SYNTHESIS OF CONTROL SYSTEMS FOR MULTIDIMENSIONAL DYNAMIC OBJECTS UNDER PARAMETRIC UNCERTAINTY BASED ON THE INVARIANCE THEORY

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Abstract: The problems of synthesis of control systems for multidimensional dynamic objects in the conditions of parametric uncertainty which are based on invariance theory are considered. When constructing stable synthesis algorithms, a recurrent method for determining pseudo-solutions of linear algebraic systems of equations is used. Computational algorithms for the solution based on singular matrix expansions are given. These computational procedures allow us to regularize the problem of synthesis of the considered algorithms for the synthesis of control systems for multidimensional dynamic objects under conditions of parametric uncertainty and improve the quality indicators of control processes.

Keywords: invariance, parametric uncertainty, multidimensional object, correcting contour, pseudo-solution, recurrent algorithm, singular matrix decomposition.

Аннотация: Инвариантлик назарияси асосида параметрик ноаниқлик шароитидаги кўп ўлчамли динамик объектларни бошқариш тизимларини синтезлаш масалалари кўриб чиқилган. Турғун синтезлаш алгоритмларини қуришда чизиқли алгебраик тенгламалар тизимларининг мавхум-ечимларини аниқлашда рекуррент усуллардан фойдаланилган. Матрицани сингуляр ёйилмаси асосида ечимни ҳисоблаш алгоритмлари келтирилган. Келтирилган ҳисоблаш амаллари кўриб чиқилаётган параметрик ноаниқлик шароитида кўп ўлчамли динамик объектларни бошқариш тизимларини синтезлаш алгоритмларини мунтазамлаш ва бошқариш жараёнларининг сифат кўрсаткичларини яхшилаш имконини беради.

Таянч сўзлар: инвариантлик, параметрик ноаниқлик, кўп ўлчамли объект, тўғирловчи контур, мавхум-ечим, рекуррент алгоритм, матрицанинг сингуляр ёйилмаси.

Аннотация: Рассматриваются вопросы синтеза систем управления многомерными динамическими объектами в условиях параметрической неопределенности на основе теории инвариантности. При построении устойчивых алгоритмов синтеза используется рекуррентный метод определения псевдорешений линейных алгебраических систем уравнений. Приводятся вычислительные алгоритмы решения, основанные на сингулярных разложениях матрицы. Приведенные вычислительные процедуры позволяют регуляризировать задачу рассматриваемых алгоритмов синтеза систем управления многомерными динамическими объектами в условиях параметрической неопределенности и повысить качественные показатели процессов управления.

Ключевые слова: инвариантность, параметрическая неопределенность, многомерный объект, корректирующий контур, псевдорешение, рекуррентный алгоритм, сингулярное разложение матрицы.

Introduction

Consider the synthesis problem for continuous dynamic objects. The main circuit of the control system includes a number of pre-defined devices that are functionally necessary for solving the control problem. These devices include an object, an actuator, and sensors.

The control object and the actuator are usually described by nonlinear differential equations. The most common approach to the synthesis of the main contour is the synthesis based on linearized equations of motion. In this case, the structure of the regulator is selected, which allows to provide a solution to be provided to the problem of adaptation, optimality, etc. It is advisable to take into account the nonlinear characteristics of the object and other elements of the system at the second stage of synthesis in order to clarify and adjust the parameters of the controller.

It is advisable to divide the synthesis of the main contour of an adaptive system into two tasks [1-5]. The first task is to synthesize a generalized configurable object that includes the object itself,

sensors, actuators, and correction devices with tunable coefficients. It is necessary to determine the laws of adjustment of these coefficients from the condition that the generalized configurable object is described by equations with constant coefficients or, more generally, equations with the desired laws of change of coefficients. The second task is to synthesize control for a generalized configurable object, i.e. for an object, usually with constant coefficients. In this case, the traditional problem of controlling a linear stationary object is solved. Fig. shows a block diagram of the adaptive control system.
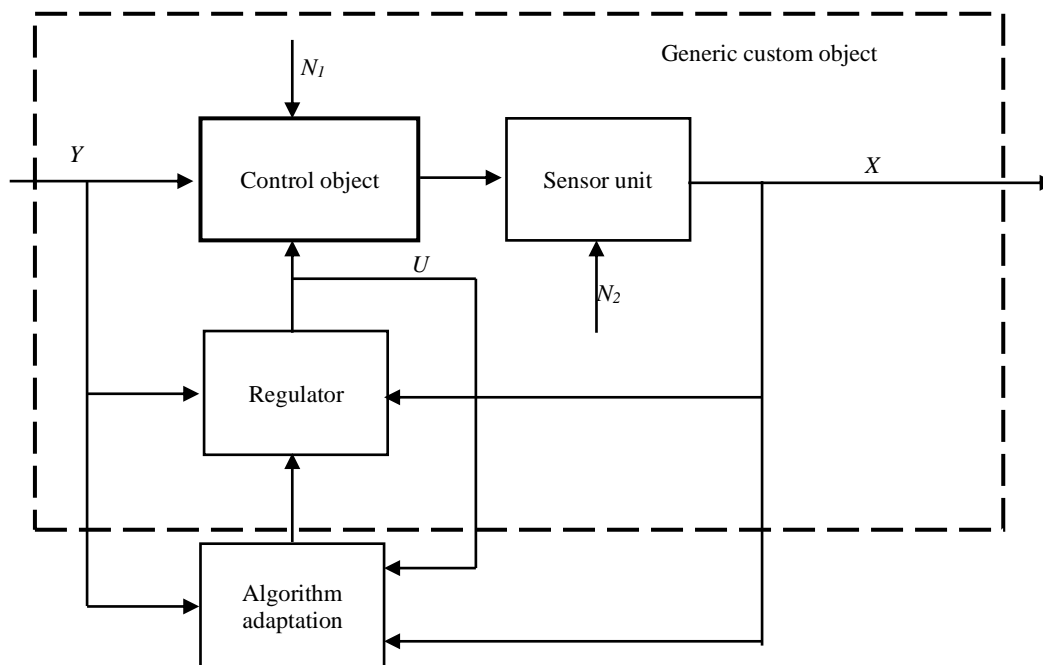

Fig. Structural scheme of adaptive control system.

In general, corrective devices can also be non-stationary. Their coefficients can be adjusted to obtain, for example, the optimal operator of the main contour when changing the statistical properties of the effects g and f. If the nature of this change is not known in advance, then the adaptation principles can be applied to adjust the correction devices accordingly [3-6]. In this case, the system will have two adaptation subsystems, of which one subsystem will solve control issues related to the non – stationary nature of the source object, and the second one will solve the optimization problem taking into account the current spectral properties of external influences.

The selection of a generalized custom object is a methodological technique in which the procedure for synthesizing an adaptive optimal system is the simplest. Structurally, of course, the correction contours of a generalized custom object can be combined with the correction contours that form the control of a generalized object [5-9].

Synthesis based on invariance

Let a multidimensional nonstationary object be together with actuators be described by an equation in matrix form

$$\dot{x} = A(t)x + D(t)\mu + C(t)f , \tag{1}$$

where $x = [x_1\, x_2\, ... x_n]^T$ – state vector; $\mu = [\mu_1\, \mu_2\, ... \mu_m]^T$ – vector of input coordinates of actuators; $f = [f_1\, f_2\, ... f_r]^T$ – vector of disturbing influences; $A(t)$, $D(t)$, $C(t)$ – matrices with variable coefficients of size $n \times n$, $n \times m$ and $n \times r$, respectively.

You need to synthesize a generalized custom object from a condition so that it is described by a stationary equation

$$\dot{x} = A^0 x + B^0 u , \tag{2}$$

where $u = [u_1\ u_2\ ...u_m]^T$ – control vector; A^0, B^0 – stationary matrices of size $n \times n$ and $n \times m$, whose coefficients correspond to the nominal parameters of the source object or are selected at the control synthesis stage u. We assume that (2) corresponds to a stable, fully controlled, and fully observable motion.

Let's represent matrices $A(t)$, $D(t)$, $C(t)$ as [1,4,7,8]:

$$A(t) = A^0 + \Delta A(t),\ D(t) = D^0 + \Delta D(t),\ C(t) = \Delta C(t), \tag{3}$$

where D^0 – a constant matrix of size $n \times m$, whose coefficients correspond to the nominal mode of operation of the object; $\Delta A(t)$, $\Delta D(t)$, $\Delta C(t)$ will be considered as a matrix of parametric perturbations.

Then, to solve the problem, assuming that the matrix $\left(I + D^{0+}\Delta N\right)$ is non – singular (I – is a unit matrix), the equation for correcting contours of a generalized configurable object can be chosen in the form [4,6,8]:

$$\mu = D^{0+}\left(D_1^0 u - \Delta K x - \Delta N \mu - \Delta R f\right), \tag{4}$$

where D^{0+} – is a pseudo-inverse matrix of size $m \times n$ for D^0; D_1^0 – is a constant matrix of size $n \times m$, whose coefficients are subject to selection; ΔK, ΔN, ΔR is a matrix of tunable coefficients of size $n \times n$, $n \times m$, $n \times r$, respectively.

From the conditions of parametric invariance and invariance with respect to f of the system (1), (4) (when these conditions are met, it is described by equation (2)), we have

$$\begin{aligned} D^0 D^{0+}\Delta K \equiv \Delta A(t), \quad D^0 D^{0+}\Delta N \equiv \Delta D(t), \\ D^0 D^{0+}\Delta R \equiv \Delta C(t), \quad D^0 D^{0+}D_1^0 = B^0. \end{aligned} \tag{5}$$

Thus, equations (4) and (5) determine the structure of the correction contours of the generalized configurable object, and the adaptation task is to adjust the coefficients ΔK, ΔN, ΔR in order to ensure the conditions (5) with proper accuracy.

If D^0 has a rank of $r_D = n$, then D^{0+} can be selected

$$D^{0+} = D^{0T}\left(D^0 D^{0T}\right)^{-1}, \tag{6}$$

where $D^0 D^{0T}$ – non-singular, i.e. it has an inverse matrix. Then $D^0 D^{0+} = I$ and equations (5) will take the form:

$$\Delta K \equiv \Delta A(t),\ \Delta N \equiv \Delta D(t),\ \Delta R = \Delta C(t),\ D_1^0 = B^0.$$

However, this is only possible at $n \leq m$. In practice, we usually have $m \leq n$. In this case, it is advisable to switch to coordinate-parametric control of the object [4,7,8,10].

Construction of stable synthesis algorithms using a recurrent method for determining pseudo-solutions of linear algebraic systems of equations

When solving system (6), computational difficulties may arise. Because of the system (6) may be ill-conditioned. In such cases, it is advisable to use stable methods for solving such systems [11-14].

In a number of computational problems in linear algebra, it is necessary to find the Moore-Penrose pseudo-inverse matrix D^{0+} with respect to the matrix D^0. The problems of numerical determination of pseudo-inverse matrices are discussed in the extensive literature [12,14-19]. One of the well-known and most common methods for finding a pseudo-inverse matrix is to use recurrent algorithms that generalize the recurrent method for determining pseudo-solutions of linear algebraic systems [12-14, 20].

In (6), the matrix D^0 is an under defined matrix of size $n \times m$ and has rank $r_D \leq n$. For this reason, we need to convert fill the matrix to a square matrix, by adding m-n zero rows to the D^0 matrix. the Machine program can perform zero-edging implicitly, so that no real storage of zero rows or arithmetic operations with zero elements are required [12,14]. Let's introduce a block matrix $F = F_{(m \times m)}$:

$$Q = \begin{bmatrix} D^0 \\ \hline 0 \end{bmatrix},$$

where $0 = 0_{[(m-n)\times m]}$ – null matrix.

The matrix Q is a square. Taking this into account, we introduce the following notation. Let q_t, $t = 1,2,...,n$ be the rows of matrix Q, and γ_{t+1}, $t = 1,2,...,n$ be a sequence of matrices with dimensions $(n \times n)$ satisfying the recurrent equation [12,20-21]:

$$\gamma_{t+1} = \gamma_t - \gamma_t q_{t+1}^T \left(q_{t+1} \gamma_t q_{t+1}^T \right)^+ q_{t+1} \gamma_t, \quad \gamma_0 = I, \tag{7}$$

$I = I_{(n\times n)}$ – identity matrix.

Denote by c_t, $t = 1,2,...,n$, the rows of the matrix $I - \gamma_n$ and set the sequences of matrices X_t, $t = 1,2,...,n$, dimension $(n \times n)$ using recurrent equations

$$X_{t+1} = X_t + \gamma_t q_{t+1}^T \left(q_{t+1} \gamma_t q_{t+1}^T \right)^+ \left(c_{t+1} - q_{t+1} X_t \right), \quad X_0 = 0, \tag{8}$$

where $0 = 0_{(n\times n)}$ - null matrix.

Systems of equations (7), (8) have the following solutions:

$$\gamma_n = I - Q_k^+ Q_k, \quad X_n = Q_k^+ (I - \gamma_n).$$

By the property of pseudo-inverse matrices [12,14-19]

$$I - \gamma_n = (Q_k^T)^+ Q_k^T = (Q_k Q_k^+)^T = Q_k Q_k^+.$$

Therefore,

$$X_n = Q_k^+ Q_k Q_k^+ = Q_k^+.$$

When solving systems of recurrent equations (7), (8), it is necessary to calculate the discontinuous function

$$\left(q_{t+1} \gamma_t q_{t+1}^T \right)^+ = \begin{cases} 1/q_{t+1}\gamma_t q_{t+1}^T, & q_{t+1}\gamma_t q_{t+1}^T \neq 0, \\ 0, & q_{t+1}\gamma_t q_{t+1}^T = 0. \end{cases} \tag{9}$$

an error in the calculation of which, when the value $q_{t+1}\gamma_t q_{t+1}^T$ is close to zero, can greatly vary distort the results obtained. In [12, 14], it was shown that the accuracy of calculations can be controlled by the traces of the matrix γ_t, $t = 1,2,...,n$ and refuse calculations if their accuracy does not correspond to the conditionality of the matrix Q.

The system of recurrent equations (7) and (8) is solved sequentially, and the values are checked

$$S_p \frac{\gamma_t q_{t+1}^T q_{t+1} \gamma_t}{q_{t+1} \gamma_t q_{t+1}^T} = 1, \quad \sum_{i,j=1}^{n} [\gamma_{ij}^2(t) - \gamma_{ij}^2(t+1)] = \|\gamma_t\|^2 - \|\gamma_{t+1}\|^2,$$

$$\rho_t = \|\gamma_t - \gamma_t^2\| / \|\gamma_t^2\| = 0.$$

The difference between these parameters and the specified ones indicates the accumulation of errors in the calculation of x_t and γ_t. If it turns out that a string with the number τ leads to large errors in $S_p\left(\gamma_{\tau-1} q_\tau^T q_\tau \gamma_{\tau-1} / q_{\tau-1} \gamma_\tau q_{\tau-1}^*\right)$, $\|\gamma_{\tau-1}\|^2 - \|\gamma_\tau\|^2$, ρ_τ, then it is natural to declare this string as a linear combination of strings $q_1,...,q_{\tau-1}$, putting $q_\tau \gamma_{\tau-1} q_\tau^T = 0$, since in this case $q_\tau \gamma_{\tau-1} q_\tau^T$ is always a sufficiently small value.

Very often the pseudo inverse matrix Q^+ is defined using limit relations:

$$D^{0+} = \lim_{\alpha \to 0} D^{0T} \left(D^0 D^{0T} + \alpha I \right)^{-1}.$$

Pre-limit matrices $D^{0T}\left(D^0D^{0T}+\alpha I\right)^{-1}$ can also be calculated using recurrent equations of type (7), (8), without resorting, for example, to the linear equation

$$X\left(D^0D^{0T}+\alpha I\right)=D^{0T},\quad \alpha>0,$$

computational algorithms for solving which are based on singular matrix expansions [12,14-17] or are iterative [11, 18-22].

To do this, in the case of matrix $D^0_{(n\times m)}$, $n<m$, we consider two sequences of matrices X^α_t, γ^α_t, $t=1,2,...,m$ defined by the following recurrent equations at $\alpha>0$ [12]:

$$\gamma^\alpha_{t+1}=\gamma^\alpha_t-\frac{\gamma^\alpha_t d^{0T}_{t+1}d^0_{t+1}\gamma^\alpha_t}{\alpha+d^0_{t+1}\gamma^\alpha_t d^{0T}_{t+1}},\quad \gamma^\alpha_0=I,$$

$$X^\alpha_{t+1}=X^\alpha_t+\frac{\gamma^\alpha_t d^0_{t+1}}{\alpha+d^0_{t+1}\gamma^\alpha_t d^{0T}_{t+1}}\left(i_{t+1}-d^0_{t+1}X^\alpha_t\right),\quad X^\alpha_0=0,$$

where $0=0_{(n\times m)}$, i_t, $t=1,2,....,m$ – rows of a unit matrix of size $n\times n$, d^0_t, $t=1,2,....,m$ – rows D^0.

Conclusion

The described way of providing a generalized configurable object the desired dynamic properties requires a fairly large amount of a priori and current information: knowledge of the form (order) of equations is required, and real-time multiparametric identification is required to compensate for parametric perturbations. In addition, it should be taken into account that full compensation of coordinate disturbances may require high speed and high power. In this case, (3) matrix $C(t)$ should be presented in a $C(t)=C^0+\Delta C(t)$, where the C^0 – component, which is determined by a valid measure of the impact of perturbation $f(t)$ on the system's output, $\Delta C(t)$ – component, which is compensated by means of parametric adjustment. The algorithms of the tuning circuits do not change [4-8].

These computational procedures allow us to regularize the problem of synthesizing a control system for multidimensional dynamic objects in conditions of parametric uncertainty based on the invariance principle and improve the quality indicators of control processes.

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