SUSTAINABLE ALGORITHMS FOR SYNTHESIS OF REGULATORS IN ADAPTIVE CONTROL SYSTEMS OF PARAMETRICALLY UNCERTAIN OBJECTS

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Abstract: The problems of formation and construction of regularized algorithms for estimating the parameters of regulators in systems of indirect adaptive control in parametrically indeterminate objects are considered. When constructing regularized estimates for regulator settings, the concept of pseudoinversion is used based on computing solutions with an unbiased square of length using an equivalent extended joint system. Algorithms for iterative refinement of the desired solution are given. These computational procedures make it possible to regularize the synthesis problem of the considered algorithms for estimating the parameters of regulators and to raise the qualitative indices of control processes of dynamic objects under conditions of parametric uncertainty.

Keywords: parametrically indeterminate objects, indirect adaptive control, regulator parameters, pseudoinversion, stable estimation algorithms.

Аннотация: Параметрик ноаниқ объектларни билвосита адаптив бошқариш тизимларида ростлагич параметрларини баҳолашнинг мунтазамлаштирилган алгоритмларини шакллантириш ва қуриш муаммолари кўриб чиқилган. Ростлагич параметрларини созлаш учун мунтазамлашган баҳони қуришда, эквивалент кенгайтирилган қушма тизимдан фойдаланиб силжимаган квадратик узунликдаги ечимларни ҳисоблаш асосидаги псевдомуносабат концепцияси қўлланилган. Қидирилаётган ечимни итератив аниқлаш алгоритмлари келтирилган. Келтирилган ҳисоблаш процедуралари ростлагич параметрларини баҳолаш алгоритмларини синтезлаш масаласини мунтазамлашга ва параметрик ноаниқлик шароитида динамик объектларни бошқариш жараёнларининг сифат кўрсаткичларини яхшилаш имконини беради.

Таянч сўзлар: параметрик ноаниқ объектлар, билвосита адаптив бошқарув, ростлагич параметрлари, псевдомуносабат, тургун баҳолаш алгоритлари.

Аннотация: Рассматриваются вопросы формирования и построения регуляризованных алгоритмов оценивания параметров регуляторов в системах непрямого адаптивного управления параметрически неопределенными объектами. При построении регуляризованных оценок для параметров настроек регуляторов используется концепция псевдообращения на основе вычисления решений с несмещенным квадратом длины с использованием эквивалентной расширенной совместной системы. Приводятся алгоритмы итерационного уточнения искомого решения. Приведенные вычислительные процедуры позволяют регуляризировать задачу синтеза рассматриваемых алгоритмов оценивания параметров регуляторов и повысить качественные показатели процессов управления динамическими объектами в условиях параметрической неопределенности.

Ключевые слова: параметрически неопределенные объекты, непрямое адаптивное управление, параметры регулятора, псевдообращение, устойчивые алгоритмы оценивания.

Introduction

High requirements imposed on the quality of functioning of modern technical systems lead to the need to develop adaptive control methods that allow to optimize control processes, ensure the operation of the control system when changing the static and dynamic characteristics of the facility, and improve the reliability of its operation [1-5].

When implementing various principles of adaptive control of an object, the question arises as to how to choose the structure of regulators of coordinate and parametric control and adaptation devices that change the parameters of regulators and the observer. Among the classes of adaptive control

systems, its practical effectiveness is distinguished by a class based on the principle of indirect adaptive control with a reference model based on the identification of parameters of the internal loop and the adjustment of controller parameters based on static algorithms [1,3].

Formulation of the problem

Consider a linear indeterminate stochastic control object of the form:

$$x_{k+1} = A_0 x_k + B_0 u_k + \Gamma_k w_k, \tag{1}$$

$$y_k = H x_k + D_k v_k,$$

where x, y, u - vectors of the state of the object, the output of the object and control; $x \in R^n$, $y, u \in R^m$; $\{w_k\}, \{v_k\}$, mutually independent white noise Gaussian sequences, $w, v \in R^m$, for which $M[w_k] \equiv M[v_k] \equiv 0$, $M\{w_k w_k^T\} = I_m$, $M\{v_k v_k^T\} = I_m$, $M[x_0] = 0$, $M[x_0 x_0^T] = P_0$; A_0, B_0 – unknown matrices.

Quite often in the theory and practice of managing dynamic objects, a control algorithm is used based on the optimal adaptive controller with a reference model [1]:

$$u_k = u_{\text{oc}k} + \xi_k, \quad u_{\text{oc}k} = -\hat{K}_k \hat{x}_k, \tag{2}$$

where u_{oc} – optimal control, formed in accordance with the principle of separation, ξ – measurable sequence of the random numbers.

The equation of state of the inner contour can be written in the form:

$$x_{k+1} = \left[A^0 - B^0 R_{1,k}\right] x_k + \left[B^0 + B^0 R_{2,k}\right] u_k + \Gamma_k w_k.$$

We will assume that the necessary and sufficient conditions for the complete adaptability of the inner contour are observed.

Consider the problem of synthesizing the optimal adaptive controller with respect to the loss functional of the form:

$$J(x,u) = E\left\{x_N^T F x_N + \sum_{k=0}^{N-1}\left[x_k^T L x_k + u_k^T K_k^{-1} u_k\right]\right\},$$

where the matrices F, L, K_k^{-1} – are symmetric, $F \geq 0, L \geq 0, K_k^{-1} > 0$.

The solution of this problem on the basis of the separation theorem is the control algorithm (2), in which estimates of the state of the inner contour are obtained using the Kalman reference filter of the form [1,3,5]:

$$\hat{x}_{k+1} = A^0 \hat{x}_k + B^0 u_k + \tilde{K}_k\left[y_k - H\hat{x}_k\right],$$

$$\tilde{K}_k = A^0 P_k H^T\left[H P_k H^T + D_k D_k^T\right]^{-1},$$

$$P_{k+1} = A^0 P_k \left(A^0\right)^T + \Gamma_k \Gamma_k^T - \tilde{K}_k[H P_k H^T + D_k D_k^T]K_k^T.$$

The reference model, embedded in the Kalman filter, is determined by the equation:

$$x_{k+1}^0 = A^0 x_k^0 + B^0 u_k, \quad y_k^0 = H x_k^0.$$

The amplification matrices $\hat{K}$ in the control law (2) are formed using the Riccati equation, into which the matrices of the parameters of the reference model:

$$\hat{K}_k = \left[K_k^{-1} + \left(B^0\right)^T S_{k+1} B^0\right]^{-1} B^{0T} S_{k+1} A^0,$$

$$S_k = \left[A^0 - B^0 \hat{K}_k\right]^T S_{k+1} A^0 + L, \quad S_N = F.$$

Suppose that the parameters of the control object (1) A_0 and B_0 have the form $A_0 = A^0 + A_k$, $B_0 = B^0 + B_k$, where A^0, B^0 – are the reference values of the parameters at which the object has the desired transient characteristics in the absence of parametric perturbations; A_k, B_k – quasi-stationary unknown matrices of perturbed parameters of the object.

The internal loop adaptation regulator is a two-level regulator, with dynamic adaptation algorithms used for calculating the intermediate matrices of the parametric mismatches of the inner contour A_k and B_k at the upper level, and static algorithms for calculating the parameters matrices of the parameters of the settings R_1 and R_2 of the internal and external loop feedback [1,3].

Here algorithms for adaptation of the upper level:

$$A_{k+1} = A_k + \varepsilon_k \, \overline{P} z_k \hat{x}_k^T,$$

$$A_{k+1} = B_k + \varepsilon_k \overline{P} z_k \, \hat{u}_k^T,$$

where the values of ε_k, $\overline{P}$, z_k, $\hat{x}_k$ are calculated on the basis of certain expressions [1].

The algorithms for adaptation of the lower level have the form:

$$B^0 R_{1,k} = A_k, \tag{3}$$

$$B^0 R_{2,k} = B_k. \tag{4}$$

In the practical implementation of algorithms for calculating the parameter settings matrices of regulators $R_{1,k}$ and $R_{2,k}$ in expressions (3), it is necessary to replace the unknown matrix B_0 by the reference matrix B^0.

Solution

When solving equations (3) and (4), we will use the statistical form of the discrepancy principle [6-12], which guarantees obtaining optimal regularized estimates of the solutions of approximate degenerate or ill-conditioned stochastic systems of linear algebraic equations on a finite sample.

In equations (3) and (4), matrix B^0 may be poorly conditioned. This circumstance necessitates the use of regularization methods [7,13-20]. Below is a regular algorithm for solving equation (3). The same algorithm can be used to solve equation (4).

We assume that the elements of the vector $a_{k,j}$ of the matrix A_k are known with errors, i.e. instead of $a_{k,j}$ its random realization $\tilde{a}_{k,j} = a_{k,j} + \Delta a_{k,j}$ is given, where the elements $\Delta a_{k,j,i}$ of vector $\Delta a_{k,j}$ almost surely satisfy conditions:

$$M(\Delta a_{k,j,i} \mid \mathsf{F}_{k,i-1}) = 0,$$

$$D(\Delta a_{k,j,i} \mid \mathsf{F}_{k,i-1}) = \sigma^2 < \infty,$$

where $\mathsf{F}_{k,i} = \sigma\{\Delta a_{k,j,1},\ldots,\Delta a_{k,j,i}\}$, $i,j=1,2,\ldots,n$; $\mathsf{F}_{k,0} = \varnothing$.

As for the matrix operator B^0, we will assume that it is given exactly, since it characterizes the reference values of the parameters of the matrix B^0.

For the solution of system (3) it is expedient to use the method of least squares, in accordance with which the estimates $\hat{r}_{1,k,j}$ of the unknown normal pseudosolution $r_{1,k,j}^* = (B^0)^+ a_{k,j}$ of system (3) are defined as the solution of problem $\inf \left\| r_{1,k,j} \right\|^2$ by $R_{1,k} = \left\{ r_{1,k,j} : Q(r_{1,k,j}) = Q_{\min} \right\}$, here:

$$Q_{\min} = \inf_{r_{1,k,j} \in R^m} Q(r_{1,k,j}),$$

where $Q(r_{1,k,j}) = \left\| B^0 r_{1,k,j} - \tilde{a}_{k,j} \right\|^2$, $\|\cdot\|$ – is the euclidean norm, $(B^0)^+$ – is the pseudoinverse matrix to B^0; $r_{1,k,j}$ – j-th column of the matrix $R_{1,k}$, $j=1,2,\ldots,n$.

Estimates $\hat{r}_{1,k,j}$ are best in the class of linear unbiased estimators [6-10]. However, in the case of a degenerate or ill-conditioned matrix B^0, they turn out to be unstable [6], i.e. $M\left\| \hat{r}_{1,k,j} \right\|^2 >> \left\| r_{1,k,j}^* \right\|^2$.

This circumstance indicates the necessity for the solution of equation (3) to use the class of estimates with an unbiased square of length, i.e. Class of estimates $\overline{R}_{1,k} = \left\{ \overline{r}_{1,k,j} : M \left\| \overline{r}_{1,k,j} \right\|^2 = \left\| r_{1,k,j}^* \right\|^2 \right\}$ [7], in this case:

$$MQ_{\min} = (n-k)\sigma^2 < \inf_{\eta_{1,k,j} \in R^m} MQ(r_{1,k,j}) = MQ(r_{1,k,j}^*) = n\sigma^2, \tag{5}$$

where $Q_{\min} = Q(\hat{r}_{1,k,j})$, $k = rankA$.

Then, on the basis of (5), it is natural to define regularized estimates of $\tilde{r}_{1,k,j}$ so that condition

$$MQ(\tilde{r}_{1,k,j}) = n\sigma^2.$$

This can be achieved if the parameter α is computed from the discrepancy equation

$$Q(r_{1,k,j}) = Q_{\min} + \delta, \quad \delta > 0.$$

Then the corresponding regularized estimates of solutions will be determined from the joint solution of the equations [6,7]:

$$(B^0)^T B^0 r_{1,k,j}^\alpha + \alpha r_{1,k,j}^\alpha = (B^0)^T \tilde{a}_{k,j}, \quad \left\| B^0 r_{1,k,j}^\alpha - \tilde{a}_{k,j} \right\|^2 = Q_{\min} + \delta.$$

Undoubted interest from a practical point of view is the case when the variance σ^2 itself is known not a priori, but only some estimate of it $\hat{\sigma}^2$. An estimate $\hat{\sigma}^2$ satisfying the conditions for the existence and uniqueness of the solution can be calculated, for example, if there is some second independent of $\tilde{\tilde{a}}_{k,j}$ random sample implementation of the right side of $\tilde{a}_{k,j}$. Then it is known [6,7], an unbiased estimate of the dispersion $\breve{\sigma}^2$ can be calculated by the formula

$$\breve{\sigma}^2 = (n-k)^{-1} \left\| \tilde{\tilde{a}}_{k,j} - B^0 \breve{r}_{1,k,j} \right\|^2, \qquad \breve{r}_{1,k,j} = B^0 + \tilde{a}_{k,j}.$$

For the practical calculation of $Q_{\min}$ and $\hat{\sigma}^2$, one can use the approach [6,7,10], which uses the equivalence of problem $B^0 r_{1,k,j} \cong \tilde{a}_{k,j}$ and systems of linear algebraic equations:

$$\left\| \begin{array}{c:c} I_n & B^0 \\ \hdashline (B^0)^T & 0_{m \times m} \end{array} \right\| \left\| \begin{array}{c} \eta \\ \hdashline r_{1,k,j} \end{array} \right\| = \left\| \begin{array}{c} \tilde{a}_{k,j} \\ \hdashline 0 \end{array} \right\|. \tag{6}$$

Then $Q_{\min} = \left\| \hat{\eta} \right\|^2$, where $\hat{\eta}$ – is the solution of the joint system of linear algebraic equations (6).

Similarly, it can be computed

$$\breve{\sigma}^2 = (n-k)^{-1} \left\| \tilde{\eta} \right\|^2,$$

where $\tilde{\eta}$ – solution of a joint system of linear algebraic equations.

$$\left\| \begin{array}{c:c} I_n & B^0 \\ \hdashline (B^0)^T & 0_{m \times m} \end{array} \right\| \left\| \begin{array}{c} \eta \\ \hdashline r_{1,k,j} \end{array} \right\| = \left\| \begin{array}{c} \tilde{\tilde{a}}_{k,j} \\ \hdashline 0 \end{array} \right\|.$$

If $rankB^0 = m$, then for the solution (6) we apply the following iteration refinement process [6,7,10,11]: We assume $\eta^{(0)} = 0$, $r_{1,k,j}^{(0)} = 0$. Iteration with number r consists of three steps:

Computes the discrepancies

$$\begin{bmatrix} f^{(r)} \\ \hdashline g^{(r)} \end{bmatrix} = \begin{bmatrix} \tilde{a}_{k,j} \\ \hdashline 0 \end{bmatrix} - \begin{bmatrix} I_n & B^0 \\ \hdashline (B^0)^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta^{(r)} \\ \hdashline r_{1,k,j}^{(r)} \end{bmatrix}; \tag{7}$$

The corrections $\delta \eta^{(r)}$, $\delta r_{1,k,j}^{(r)}$ are determined by solving the system

$$\begin{bmatrix} I_n & B^0 \\ (B^0)^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta\eta^{(r)} \\ \delta r_{1,k,j}^{(r)} \end{bmatrix} = \begin{bmatrix} f^{(r)} \\ g^{(r)} \end{bmatrix}; \tag{8}$$

There are new approximations $\eta^{(r+1)} = \eta^{(r)} + \delta\eta^{(r)}$, $r_{1,k,j}^{(r+1)} = r_{1,k,j}^{(r)} + \delta r_{1,k,j}^{(r)}$.

Note that for $r = 0$ from formula (7) it follows that $f^{(0)} = \tilde{a}_{k,j}$, $g^{(0)} = 0$, and then (8) turns into (6). Thus, the first iteration is simply the original solution of the original problem $B^0 r_{1,k,j} \cong \tilde{a}_{k,j}$. If $rankB^0 < m$, then the process of iterative refinement of a pseudo-solution of a system of linear equations is given in [13,14].

As is known [13-19], in the case of algorithms for adapting the lower level (3) it has the form:

$$R_{1,k} = B^{0^+} A_k, \tag{9}$$

where B^{0^+} – matrix pseudo-inverse to matrix B^0.

If the columns of the matrix B^0 are linearly independent (i.e., the core of this matrix is trivial: $N(B^0) = \{0\}$), then the estimate $R_{1,k}$ minimizes the quadratic form $\left\| A_k - B^0 R_{1,k} \right\|^2$, while the columns B^0 are linearly dependent, then $R_{1,k}$ is the estimate with the minimum norm among all estimates that minimize this form. $R_{1,k}$ covariance matrix:

$$Q = \mathrm{var}\{R_{1,k}\} = \left(B^{0^T} B^0 \right)^+.$$

If columns B^0 are linearly independent, then

$$B^{0^+} = \left(B^{0^T} B^0 \right)^{-1} B^{0^T}, \quad Q = \left(B^{0^T} B^0 \right)^{-1}.$$

In expression (9) for the formation of control law $R_{1,k}$, there is a pseudoinverse matrix B^{0^+}. It is clear that the quality of control processes of the synthesized adaptive control system substantially depends on the accuracy of control definition (9). In view of this circumstance, it becomes necessary to use effective pseudoinverse algorithms for overdetermined matrices.

It is known [13,14, 17-19] that the task of calculating a pseudoinverse matrix is generally unstable with respect to errors in the task of the initial matrix. Moreover, the errors of the initial data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of adequacy of the model to the real process. The influence of rounding errors produced during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed on the basis of well-known analysis methods and accuracy balances.

Typically, industrial processes are characterized by a time correlation of the noise acting on the object. Under these conditions, the statement about the uncorrelated noise and regressors, that is, the observed coordinates, is unfair. The correlation of noise when using the least squares method causes a shift in the parameter estimates, an increase in the variance of these estimates. To obtain unbiased estimates, you can use the generalized least squares method [13,19]:

$$R_{1,k} = V^{-1} B^{0^T} U^{-1} A_k, \tag{10}$$

where $V = B^{0^T} U^{-1} B^0$.

The estimate (10) does not always exist. For its validity, it is necessary that the matrices U^{-1} and V^{-1} exist. In the case when the matrix U^{-1} exists and the matrix V^{-1} does not exist, generalization (10) is the formula

$$\hat{R}_{1,k} = V^+ B^{0^T} U^{-1} A_k,$$

where V^+ – pseudo-inverse to V matrix [13,14].

Following [13], it can be shown that the optimal in the mean square estimate of the vector $R_{1,k}$ with the degeneracy of the matrix U is:

$$\hat{R}_{1,k} = \overline{R}_{1,k} + MB_2^{0^{T(1)}}\left(B_2^{0(1)}MB_2^{0^{T(1)}}\right)^{-1}\left(A_{k,2}^{(1)} - B_2^{0(1)}\hat{R}_{1,k}\right),$$

$$R_{1,k} = MB^{0^T}U^+A_k, \quad B^{0(1)} = \left(B_1^{0^{T(1)}}, B_2^{0^{T(1)}}\right)^T, \quad A_k^{(1)} = T^T A_k = \left(A_{k,1}^{T(1)}, A_{k,2}^{T(1)}\right)^T,$$

where $M = \left(B^{0^T}U^+B^0\right)^{-1}$, T is the orthogonal matrix leading U to a diagonal view with the first p_k diagonal elements other than zero; p_k is the rank of the matrix U.

One approach to simplify the calculation of estimates of $\hat{R}_k$ by (10) is to use the following recursive procedure for finding estimates of the generalized least-squares method [13,14]. For the covariance measurement matrix, estimates of parameters R_k at the $(i+1)$-rd step of the recurrence procedure are found by the formula:

$$\hat{R}_{k,i+1} = \hat{R}_{k,i} + V_i^{-1}C_{i+1}^*\left(g_{i+1}^* - \hat{R}_{k,i}^T C_{i+1}^*\right). \tag{11}$$

Here

$$C_{i+1}^* = \left(C_{i+1} - B_i^0 U_i^{-1} s_{i+1}\right)\beta_{i+1}^{-1/2}, \quad (C_i = C(k_i));$$

$$g_{i+1}^* = (g_{i+1} - s_{i+1}^T U_i^{-1} A_{k,i})\beta_{i+1}^{-1/2}; \tag{12}$$

$$\beta_{i+1} = s_{i+1,i+1} - s_{i+1}^T R_i^{-1} s_{i+1},$$

where $s_{i+1} = (s_{i+1,1}, s_{i+1,2}, ..., s_{i+1,i})^T$ - i-dimensional vector of measurement covariance at point s_{i+1} with measurements at $k_1, k_2, ..., k_i$; $s_{i+1,i+1}$ is the dispersion value at k_{i+1}.

From the expressions (12) it can be seen that to calculate C_{i+1}^*, g_{i+1}^* and β_{i+1}, it is necessary to pre-calculate the vector $\mu_{i+1} = U_i^{-1} s_{i+1}$. In the case of the tape matrix U_i^{-1}, the vector μ_{i+1} has the form:

$$\mu_{i+1} = (0,0,...,0,\gamma_i)^T. \tag{13}$$

In view of (13), expressions (12) take a simple form:

$$C_{i+1}^* = \left(C_{i+1} - C_i\gamma_i\right)\beta_{i+1}^{-1/2}; \quad g_{i+1}^* = \left(g_{i+1} - g_i\gamma_i\right)\beta_{i+1}^{-1/2}; \quad \beta_{i+1} = s_{i+1,i+1} - s_{i+1,i}\gamma_i.$$

Conclusion

The above computational procedures make it possible to regularize the problem of synthesizing algorithms for estimating the parameters of regulators in adaptive control systems with a tunable model and to improve the quality indicators of the processes of controlling dynamic objects under conditions of parametric uncertainty.

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