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APPLICATION OF WAVELETS FOR FORMING DISCRETE SELECTIONS OF CONTINUOUS SIGNALS

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Abstract: *Abstract: The paper deals with theory questions, as well as examples of calculations based on modern basis functions with compact carriers for solving problems of forming discrete samples of continuous signals with finite energy. The method is based on the law of asymptotic attenuation of the values of the wavelet coefficient moduli to zero as n → ∞, and the speed of their motion to zero depends on the choice of the wavelet. This method can be defined as the summation of the octave energy components of the coefficients of fast wavelet transformations with the binary law of decreasing sampling steps.*

Key words: *basic function, compact carrier, sample, wavelet, wavelet transform, fast algorithm.*

Introduction

Wavelet-bases are widely used in information compression problems, when separating the signal from noise and in other applications. Just as the sampling theorems are the basis for the processes of reconstructing the signals f (t) as functions of time with a bounded spectrum from a discrete set of f (ih) samples, a similar question arises concerning wavelets. It is possible to set up in a broader sense: is the discrete set of values obtained sufficient enough to restore f as an object without the assumption of the finiteness of the signal spectrum.

Significant progress in the use of wavelets in various applications is associated, first of all, with the existence of fast algorithms for spectral discrete transformations whose class is much wider than the set of fast transformations in the basis of complex exponential functions [1,2,3,5]. To obtain discrete samples of the required length, which provides a given accuracy of reconstructing a continuous signal, the proper spectra of wavelet coefficients in different bases are used, and not spectral Fourier coefficients.

1. Construction of weight basis

The construction of wavelets for specific problems is usually optimized in the sense of obtaining coefficients that are close to zero. This depends, in the main, on the smoothness of the function f, on the number of zero moments of the wavelet ψ and on the size of its carrier. It is known that ψ has p zero moments, if

$$\int_{-\infty}^{\infty} x^k \psi(x)dx = 0,\ \ 0 \le k < p. \quad (1)$$

This means that ψ is orthogonal to any polynomial of degree p-1. The orthogonality condition for wavelets to polynomials determines their smoothness and alternating sign.

The wavelet basis is given by an iterative algorithm with a scale change and a shift of a single function. This leads to a procedure of multiple-scale analysis (KMA), which is a description of the space L2 (R) through hierarchically embedded spaces Vs that do not intersect and have the following property: for any function f (x) Vs, its compressed modification belongs to Vs-1. In addition to the sequence of these spaces, they form a system of pairwise orthogonal subspaces Ws L2, which is obtained as follows: Ws is the space added upon transition from Vs to the next larger space Vs-1 in the sequence

$$V_1 \subset V_0 \subset V_{-1} ... \subset V_S \subset V_{S-1} \subset ... \subset L^2,$$

Equality of the form

$$\varphi(x) = \sqrt{2}\sum_{k=-\infty}^{\infty} c_k \varphi(2x-k), \quad (2)$$

with the coefficient vector {ck} ε l2 (Z) is called the scaling equation and is the basis of the entire MRA process.

At the beginning of the scaling, the stretching step σ> 1 of the operator Dσ is chosen and only integer values of the operator are used, usually σ = 2. The primary wavelet ψ must satisfy the equation:

$$D_2\psi(x) = \sqrt{2}\sum_{k=0}^{n} c_k (x-k), \quad (3)$$

where c_k – coefficients calculated in the form of scalar products of type $(f,\psi_{n,k})$ и $(f,\psi_{n+1,k})$.

Assuming that

$$\psi_{s,k}(x) = 2^{-\frac{s}{2}}\psi\left(\frac{x-k2^s}{2^s}\right) = 2^{-\frac{s}{2}}\psi\left(x/2^s - k\right), \quad (4)$$

then the family of wavelet functions of the form {ψs, k} becomes an orthonormal basis.

The coefficient vector c0 of the primary wavelet uniquely determines the scaling function for each wavelet type. When performing calculations, there is no need to constantly refer to the scaling function and to the primary wavelet for calculating the coefficients of new orders. By this property the multiscale analysis differs from the Fourier analysis, where it is necessary to calculate the values of the basis function exp (-jω) under many transformations.

We transform the continuous signal f to a discrete form, i.e. we represent it as a row vector containing n real numbers fi (i = 0,1, ..., n-1). For fL2, the partial sum with the wavelet coefficients ck is interpreted as the difference between the two current approximations f (with the permissions 2-s + 1 and 2-s), and the corresponding sets of sampling points appear in the multi-scale analysis. Approximation with a resolution of 2-s contains all the necessary information for calculating the coefficient vector with a coarser resolution of 2-s-1.

The Haar basis is obtained from multi-scale piecewise-constant functions [1,4,7]. The scaling function has the form 1 [0,1]. The corresponding filter has two nonzero coefficients equal to 2-1 / 2 for n = 0 and n = 1. In [1] the expression

$$\frac{1}{\sqrt{2}}\psi\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}\left(\varphi(x-1) - \varphi(x)\right), \quad (5)$$

therefore

$$\psi(x) = \begin{cases} 1, & if \quad 0 \le x < 1, \\ -1, & if \quad 1/2 \le x < 1, \\ 0 & at \quad other\ points. \end{cases} \quad (6)$$

Wavelet Haar is the simplest representative of the Daubechies wavelet family. Their classification is based on the number of zero moments. This wavelet has only one zero moment and has received the designation Db1 in the literature. It is characterized by the smallest carrier among all orthogonal wavelets, is symmetric and the algorithm for calculating the coefficients does not contain multiplication operations. But its scaling function and the primary wavelet are discontinuous and therefore ill-suited for approximating smooth functions. The process of convergence of the algorithm of the inverse Haar wavelet transform to the function f with increasing number of iterations is very slow, which is explained by the weak damping of the frequency response, whose envelope asymptotically tends to zero according to the law | ω-1 |.

2. Octal methods of signal energy calculation

To estimate the convergence to the total energy of the signal of its spectral energy with respect to the squared coefficient vector, it is necessary to calculate the octave energy spectrum [1,6,9]. Its advantage is the property of invariance with respect to signal shifts in time, if they are stationary. This unique property, which also has spectra in the basis of complex exponential Fourier functions.

To obtain the integral value of the energy spectrum, it is necessary to calculate the sums of the squares of the orthonormal Haar wavelet coefficients taking into account the binary octave weights [1,8]:

$$E_C = \left(\left(c_0^2 + c_1^2\right) + 2\left(c_2^2 + c_3^2\right) + 2^2 \sum_{k=2^2}^{2^q-1} c_k^2 + \right.$$
$$\left. + 2^3 \sum_{k=2^3}^{2^4-1} c_k^2 + \cdots + 2^n \sum_{k=2^{n-1}}^{2^q-1} c_k^2 \right)$$

where q is the largest order of iterations used in the implementation of the fast algorithm, and binary multipliers are necessary to take into account the weights of octave orthogonal components in the total sum.

Thus, a scheme of an algorithm for successive approximations can be constructed that allows one to justify the minimum length L of a sample of discrete samples of a continuous signal f (x) that provides a predetermined error ε value of the signal energy approximation based on fast wavelet transform (WAV) algorithms. Denote the value of the spectral energy obtained at the iteration with the number s, as Es, and at the subsequent iteration with the number s + 1 - as Es + 1. Then the sign of the achievement of a given accuracy in the repetition of iterations becomes the instant of fulfillment of the inequality:

$$\left|E_{E,S+1} - E_{E,S}\right| \le E, S = 1,2,\ldots \quad (8)$$

Here one can turn to the well-known theorem of Plancherel:

Theorem 1. For each square-integrable function f (x) ε L2 (-∞, ∞), the integral

$$F_k(\omega) = \left(\frac{1}{\sqrt{2\pi}}\right) \int_{-\infty}^{\infty} f(x) \exp(-j\omega x) dx, \quad (9)$$

converges in L2 to some function F (ω) as k → ∞, that is,

$$\lim_{k\to\infty} \int_{-\infty}^{\infty} \left|F(\omega) - F_k(\omega)\right| d\omega = 0, \quad (10)$$

The problem of increasing the rate of convergence of wavelet approximations to a function and, consequently, the minimization of the number of coefficients is of great importance. It can be solved by applying the higher-order Daubechies basis functions. The complete set of functions φs, k and ψs, k for all s form an orthonormal basis L2 (R). When performing KMA, they play the role of low-frequency and high-frequency filters, which complement each other [1]. These filters have 2p zero coefficients. The carrier of the corresponding scaling function is the interval [0,2p - 1].

Daubechy wavelets of higher orders are smoother than Haar wavelets and, in addition, their modulus coefficients decrease much faster. Therefore, the speed of signal recovery during the inverse transformation and the convergence of spectral energy to the function with the addition of new octave coefficients become higher. The question of finding the wavelet form, which corresponds to the highest rate of convergence and, as a result, the smallest number of iterations, is naturally posed.

We choose from the family of orthonormal wavelets with finite support as an example the wavelet Db2 with two zero moments. The set of discrete filter elements of the primary wavelet in this case contains 4 elements. A fast wavelet transform can be realized in the form of a cascade connection of low-frequency and high-frequency filters.

The energy of the sums of the squares of the weighted coefficients, the results of the orthonormal transformation, can be determined by a formula analogous to (7), taking into account the number of elements of the primary wavelet:

$$E_\varepsilon = \left(\sum_{k=0}^{3} c_k^2 + 2\sum_{k=2^2}^{2^{3-1}} c_k^2 + \right.$$
$$\left. 2^2 \sum_{k=2^3}^{2^4-1} c_k^2 + \cdots + 2^p \sum_{k=2^{n-1}}^{2^n-1} c_k^2 \right) n \quad , (11)$$

With increasing order of continuous wavelet Daubechies, the rate of decrease of the moduli of the coefficients increases. The task of selecting the necessary vector of coefficients that provide the required value reduces to finding a specific type of wavelet that minimizes the number of iterations of the WB algorithm.

3. Example of application of the method

In the world practice, a broad front is the development of methods for processing and interpreting digital material obtained as a result of measurements of the intensities of fields of various physical nature. Let us take as the continuous signal the experimentally obtained function f (x), whose graph on the closed interval [a, b] is shown in Fig. 1. It contains 128 samples, which is almost half the sample length of the profile function of the two-dimensional field.

For the purpose of performing examples of estimating the energy of high-frequency wavelet coefficients to calculate the signal sample length, we perform calculations for two cases:

- The sampling step is h = 2 signals and the number of samples in the sample is 64;
- The sampling step is h = 1, i.e. The sample with a doubled frequency and the number of samples of the same signal is 128.

In Fig. 2. The histogram of the first half of the Haar coefficients is shown in the dyadic ordering of their numbers. It is obvious that the rate of decrease of the values of the moduli of the coefficients with increasing number is determined by the frequency characteristic of the Haar wavelet, which varies according to the law FH (ω) ~ ω-1. To increase the rate of convergence of the energy of the spectrum to the total energy as n → ∞, it is necessary to proceed to the use of wavelets of higher orders (Db3, Db4, etc.)

We apply to the function f (x) the fast transformation algorithm in the basis of the continuous wavelet functions of Daubechy db2 (p = 2).

The Fourier frequency response of such a wavelet varies according to the law Fdb2 (ω) ~ ω-2. The histogram of the first half of the vector of 128 coefficients of the expansion of the function f (x) with respect to the basis functions db2 for the dyadic ordering method is shown in Fig. 3.

Spectral energy calculations are performed in accordance with two options:

- At step h = 2 (ie 64 samples of the function and 64 wavelet coefficients, respectively), the energy value is Eε = 137.158;
- For a doubled sample rate (128 samples, step h = 1, 128 coefficients), we get Ec = 137.498. Thus, one can speak of the motion of the spectral energy of a sequence of wavelet coefficients to a certain value of the total energy.

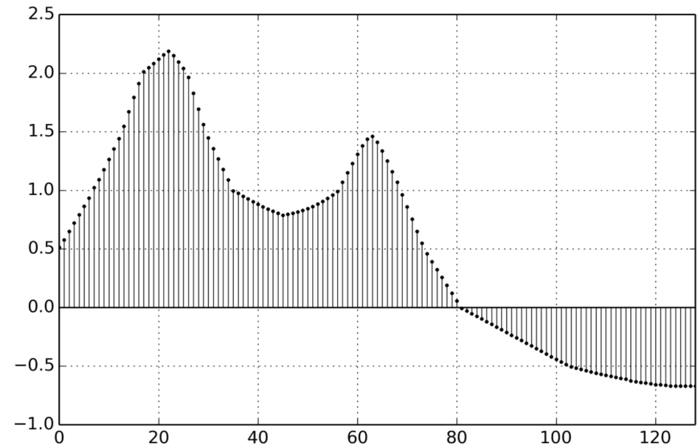

Fig.1. ***The graph of the experimentally obtained continuous signal***

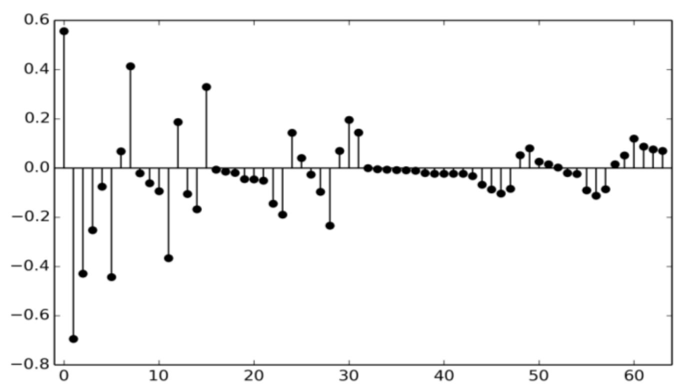

Fig. 2. ***Histogram of the first half of the Haar coefficients in the dyadic ordering of their numbers***.

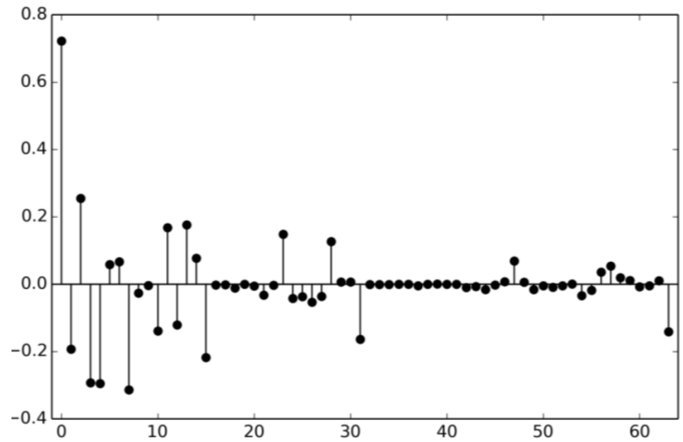

Fig. 3. ***The histogram of the first half of the vector of 128 coefficients of the expansion of the function f (x) with respect to the basis functions db2 with the dyadic ordering method***

Conclusion

Thus, significant progress in the use of wavelets in various applications is associated, first of all, with the existence of algorithms for fast spectral discrete transformations whose class is much broader than the set of fast transformations in the basis of complex exponential functions. To substantiate the necessary lengths of discrete samples of continuous signals that provide a given recovery accuracy, we use the own spectra of the Daubechies wavelet coefficients of different orders, rather than spectral Fourier coefficients.

The resulted examples of concrete signals - the functions of the geomagnetic field profiles, which, as a result of processing information about this field in the form of counts using algorithms of fast wavelet transforms and subsequent estimation of the energy (power) of the octave spectrum of wavelet coefficients, make it possible to judge the degree of approximation of the sample lengths to the required and sufficient size.

REFERENCES

1. Blatter K. Wavelet analysis. Fundamentals of the theory. M.: The technosphere. 2006. 272 p.
2. Ahmed N, Rao K. Orthogonal transformations in the processing of digital signals. Trans. with English. M.: Communication. 1980. 248 p.
3. Dremin IM, Ivanov OV, Nechitailo VA Wavelets and their use. Uspekhi Fizicheskikh Nauk. T.171. 2001. № 5. p.465-501.
4. Novikov A.K. Polyspectral analysis. - SPb.: Central Research Institute of Krylov, 2000, -162 p.
5. Svinyin S.F. Theory and methods of forming samples of signals with infinite spectra. -Spb.: Nauka, 2016. -71c.
6. Hakimjon Zayniddinov, Madhusudan Singh, Dhananjay Singh Polynomial Splines for Digital Signal and Systems (Monograph in English). LAMBERT Academic publishing, Germany, 2016 year, 208 p.
7. Zainidinov H.N. Methods and means of signal processing in piecewise polynomial bases. Monograph. Tashkent - "Fan technology" - 2014, 190 p.
8. Zaynidinov H.N., Dannanjay Singh, Hoon Jae Lee. Piecewise-quadratic Harmut basis function and their applications to problems in digital signal processing. International Journal of Communication Systems, John Wiley & Sons, Ltd., DOI: 10.1002 / dac.1093, Jan. 2010. London, SCI-E. www.interscience.wiley.com
9. S. Malla. "Wavelets in signal processing".Moscow. "Peace". 2005