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SUSTAINABLE ESTIMATION OF PARAMETERS AND COVARIATION OF DISTURBANCE VECTORS IN UNCERTAIN SYSTEMS

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Abstract: *Algorithms for the formation of a procedure for the stable estimation of parameters matrices and covariances of perturbation vectors in indefinite dynamic systems based on the concepts of matrix pseudo-inversion are given. For stable pseudo-inversion, the matrix partitioning method is used using simplified regularization. The above algorithms allow for a stable estimation of the matrix of parameters and covariances of the perturbation vectors in dynamic systems and thereby increase the accuracy of adaptive control systems operating in parametric and signal uncertainty conditions.*

Keywords: *indefinite system, system parameters, perturbation vector covariance, stable estimation, regularization.*

I. Introduction

The problem of estimating parameters and linear parametric functions in a multidimensional linear indefinite stochastic model becomes very important. The practical importance of such models is justified in [1-6], which also contains further references. The indefinite-stochastic model of observation considered in the article is very general and covers very widespread cases when models with uncertain parameters and uncertain limited disturbances contained in certain sets are investigated.

When implementing the above algorithms for solving the considered problem, situations arise when the matrix of observations for estimating model parameters can be ill-conditioned. This circumstance predetermines the need for the use of regularization methods [7-10]. Below is a regular algorithm for calculating the dynamics and control matrices in the indefinite-stochastic model of an object.

II. Formulation of the problem

Will consider the indefinite dynamic system described by the following equation:

$$x_k = Ax_{k-1} + Bu_k + \xi_k, \quad k = 1, 2, ..., \quad (1)$$

where x_k – state vector dimension - n, u_k – vector of input signals of dimension - m, ξ_k – vector of unmeasured Gaussian random perturbations with $M(\xi_k) = 0$; matrices A and B are unknown. Random vectors ε_k, $k = 1,2,...$ represent the vector white noise with $M(\varepsilon_i) = 0$ and the unknown covariance matrix V:

$$M(\varepsilon_i \varepsilon_k)^T = \delta_{ik} V \text{ for all } i, k ,$$

wherein

$$V = \sum_{i=1}^{q} v_i V_i ,$$

where $v = (v_1, ..., v_q)^T \in \mathrm{N} \subset R^q$, at that, the set N of all possible column vectors v is determined by the only condition: $V>0$.

Following [11], columns z_k of height $n+m$ will be represented by expressions

$$z_k = \begin{bmatrix} x_{k-1} \\ \hdashline u_k \end{bmatrix}, \quad k = 1,2,..., \qquad (2)$$

and make block $n \times n(n+m)$ -matrix type

$$W_k = \begin{bmatrix} z_k^T & 0 & \dots & 0 \\ 0 & z_k^T & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & z_k^T \end{bmatrix} = I_n \otimes z_k^T, \quad (3)$$

where I_n is the unit matrix of order *n*, 0 is a row of *m* zeros, and $\otimes$ – is the sign of Kronecker's multiplication of matrices, $k = 1,2,...$.

We write $nN \times n(n+m)$ -matrix W in the form:

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} I_n \otimes z_1^T \\ I_n \otimes z_1^T \\ \vdots \\ I_n \otimes z_n^T \end{bmatrix}. \qquad (4)$$

We introduce the notation: $X = \{X_1,..., X_N\}$, $\varepsilon = \{\varepsilon_1,...,\varepsilon_N\}$.

The relationship between *X* and γ is written in the form

$$X = \tilde{W}\gamma + \varepsilon,$$

where matrix $\tilde{W}$ is obtained from matrix W, defined by (2) - (4), replacing $x(k-1)$ with $X(k-1)$, $\gamma^{(i)} = \|a_{i1} \dots a_{in}\, b_{i1} \dots b_{im}\|^T$, $i = 1,..., n+m$, $\gamma^{(i)^T}$ – *i*-th row of the block matrix $[A \mathrel{\vdots} B]$.

The covariance matrix K can be represented as:

$$K = I_n \otimes V.$$

Since *V* is nondegenerate, K – is also nondegenerate and

$$K^{-1} = I_n \otimes V^{-1}. \qquad (5)$$

Based on (3) - (5) you can write

$$K^{-1}W = \begin{bmatrix} V^{-1}(I_{n+m} \otimes z_k^T) \\ V^{-1}(I_{n+m} \otimes z_k^T) \\ \vdots \\ V^{-1}(I_{n+m} \otimes z_k^T) \end{bmatrix}.$$

Then

$$V^{-1}(I_{n+m} \otimes z_k^T) = (I_m \otimes z_k^T)(V^{-1} \otimes I_{n+m}).$$

Based on (3), you can write

$$V^{-1}W_k = W_k(V^{-1} \otimes I_{n+m}).$$

From here, from (4), (5), we obtain

$$K^{-1}W = W(V^{-1} \otimes I_{n+m}),$$

and therefore

$$W^T K^{-1} = (V^{-1} \otimes I_{n+m})W^T. \qquad (6)$$

Thus, for a given matrix $V = \sum_{i=1}^{q} v_i V_i$, the maximum likelihood estimate $\hat{\gamma}$ of the parameter vector γ is obtained by minimizing γ functions

$$\Theta(\gamma) = (x - W\gamma)^T K^{-1}(x - W\gamma),$$

where $K^{-1} = \left[I_n \otimes \sum_{i=1}^{q} v_i V_i \right]^{-1}$ and, therefore [11], must satisfy the equation

$$W^T K^{-1} W\hat{\gamma} = W^T K^{-1} x.$$

Taking into account (6) you can write

$$(V^{-1} \otimes I_{n+m})W^T W\hat{\gamma} = (V^{-1} \otimes I_{n+m})W^T x.$$

But due to the nondegeneracy of the matrix $V^{-1} \otimes I_{n+m}$, the last equation is equivalent to

$$W^T W\hat{\gamma} = W^T x. \qquad (7)$$

Equation (7) with respect to estimating the vector of parameters according to (3) and (4) can be represented as [11]:

$$\sum_{k=1}^{N} z_k^T z_k \hat{\gamma}_i^T = \sum_{k=1}^{N} z_k^T x_{i,k}, \quad i = 1,..., n.$$

Considering that the lines $\gamma^{(1)^T},...,\gamma^{(N)^T}$ make up the matrix $[A \mathrel{\vdots} B]$, these equations will be reduced to the matrix equation

$$\sum_{k=1}^{N} z_k^T z_k \left[\hat{A} \mathrel{\vdots} \hat{B}\right] = \sum_{k=1}^{N} z_k^T x_{i,k}, \quad i = 1,..., n, \quad (8)$$

where z_k is defined in (2).

Entering the $k \times n$ -matrix

$$X_k = [x_1 \mathrel{\vdots} x_2 \mathrel{\vdots} \dots \mathrel{\vdots} x_k]^T, \quad k = 1,2,...,$$

and $k \times (n+m)$ - matrices

$$Z_k = \left[\begin{array}{c:c:c:c} x_0 & x_1 & \dots & x_{k-1} \\ \hdashline u_1 & u_2 & \dots & u_k \end{array}\right]^T, \quad k = 1,2,...,$$

equation (8) with respect to the estimate $\left[\hat{A} \;\vdots\; \hat{B}\right]$ we write in the form

$$Z_N^T Z_N \left[\hat{A} \;\vdots\; \hat{B}\right] = Z_N^T X_N, \qquad (9)$$

where

$$Z_N^T Z_N = \left[\begin{array}{c:c} \sum\limits_{k=1}^{N} x_{k-1} x_{k-1}^T & \sum\limits_{k=1}^{N} x_{k-1} u_k^T \\ \hdashline \sum\limits_{k=1}^{N} u_k x_{k-1}^T & \sum\limits_{k=1}^{N} u_k u_k^T \end{array}\right],$$

$$X_N Z_N^T = \left[\begin{array}{c:c} \sum\limits_{k=1}^{N} x_k x_{k-1}^T & \sum\limits_{k=1}^{N} x_k u_k^T \end{array}\right].$$

Compared with equation (7) with respect to $\hat{\gamma}$, equation (9) with respect to $\left[\hat{A} \;\vdots\; \hat{B}\right]$ has the advantage [11] that the square matrix $Z_N^T Z_N$ included in it has an order of $n+m$, n times smaller than the order of $n(n+m)$ square matrix $W^T W$ in equation (7).

III. Solution of the task

If matrix $Z_N^T Z_N$ has $n+m$ linearly independent columns, then equation (9) has a unique solution

$$\left[\hat{A} \;\vdots\; \hat{B}\right] = (Z_N^T Z_N)^{-1} Z_N^T X_N. \qquad (10)$$

Square matrix $Z_N^T Z_N$ ($\left|Z_N^T Z_N\right| \neq 0$) is divided into blocks

$$Z_N^T Z_N = \left[\begin{array}{c:c} R & U \\ \hdashline U^T & S \end{array}\right], \qquad (11)$$

where

$$R = \sum_{k=1}^{N} x_{k-1} x_{k-1}^T,$$

$$U = \sum_{k=1}^{N} x_{k-1} u_k^T,$$

$$S = \sum_{k=1}^{N} u_k u_k^T.$$

For the inversion of the matrix $Z_N^T Z_N$ in (10), it is advisable to use the Frobenius formula [12,13].

If R – is a nondegenerate square matrix ($|R| \neq 0$), you can write

$$(Z_N^T Z_N)^{-1} =$$

$$= \left[\begin{array}{c:c} R^{-1} + R^{-1} U H^{-1} U^T R^{-1} & -R^{-1} U H^{-1} \\ \hdashline -H^{-1} U^T R^{-1} & H^{-1} \end{array}\right], \qquad (12)$$

where

$$H = S - U^T R^{-1} U.$$

If $|S| \neq 0$, then we can come to the expression of the form [13]:

$$(Z_N^T Z_N)^{-1} = \left[\begin{array}{c:c} F^{-1} & -F^{-1} U S^{-1} \\ \hdashline -S^{-1} U^T F^{-1} & S^{-1} + S^{-1} U^T F^{-1} U S^{-1} \end{array}\right],$$

where

$$F = R - U S^{-1} U^T.$$

At $N < n(n+m)$, equation (9) has infinitely many solutions. We will take the decision $\left[\hat{A} \;\vdots\; \hat{B}\right]$

$$\left[\hat{A} \;\vdots\; \hat{B}\right] = (Z_N^T Z_N)^{+} Z_N^T X_N = Z_N^{+} X_N \qquad (13)$$

c minimum rate $\left\|\left[\hat{A} \;\vdots\; \hat{B}\right]\right\| = \sqrt{tr(A^T A + B^T B)}$, $rank Z_N^T Z_N < n+m$.

Direct use of relation (13) can lead to a decrease in the accuracy of estimation of matrices $\left[\hat{A} \;\vdots\; \hat{B}\right]$, since they use the pseudo-reversal operation. In the case when matrix $Z_N^T Z_N$ is a matrix of not full rank, then the problem in question is incorrectly posed. To impart numerical stability to the pseudo-inversion procedure of matrix $Z_N^T Z_N$, here it is advisable to use the concepts of regular methods [7,8,14,15].

For pseudo-matrix $Z_N^T Z_N$ in (13) will use the block partition matrices [16]. Let R in (11) be a non-singular matrix and $r_R = r_{Z_N^T Z_N}$.

Then $S = U^T R^{-1} U$ [13], and therefore

$$Z_N^T Z_N = \left[\begin{array}{c} R \\ \hdashline U^T \end{array}\right] R^{-1} (R \quad U),$$

$$Z_N^T Z_N = \left[\begin{array}{c} R \\ \hdashline U^T \end{array}\right] (I \quad R^{-1} U),$$

$$(I \quad R^{-1} U) = R^{-1} (R \quad U).$$

Therefore

$$(Z_N^T Z_N)^+ = (R \quad U)^+ (R^{-1})^+ \begin{bmatrix} R \\ \hdashline U^T \end{bmatrix}^+ =$$

$$= (R \quad U)^+ R \begin{bmatrix} R \\ \hdashline U^T \end{bmatrix}^+ .$$

Using formulas

$$U^+ = (U^T U)^{-1} U^T$$

and

$$(U^T)^+ = U(U^T U)^{-1}$$

can write

$$(Z_N^T Z_N)^+ = \begin{bmatrix} R \\ \hdashline U^T \end{bmatrix} (RR^T + UU^T)^{-1} \times \times R(R^T R + UU^T)^{-1} (R^T \quad U). \tag{14}$$

To give greater numerical stability in the implementation of the pseudo-matrix algorithm (14), it is advisable to present it in the form:

$$(Z_N^T Z_N)^+ = \begin{bmatrix} R \\ \hdashline U^T \end{bmatrix} g_\alpha(P) R g_\beta(D) (R^T \quad U), \tag{15}$$

where

$$P = RR^T + UU^T,$$
$$D = R^T R + UU^T,$$
$$g_\alpha(P) = (P + \alpha I)^{-1},$$
$$g_\beta(D) = (D + \beta I)^{-1},$$

$\alpha > 0,\ \beta > 0$ – regularization parameters.

Selection regularization parameters α and β in (15) is advisable exercise carried out on the basis of model examples models [17].

The empirical covariance matrix of the perturbation vector can be calculated based on the estimates of $\hat{A}$ and $\hat{B}$ as follows [11,18]:

$$\hat{V} = \frac{1}{N} \sum_{k=1}^{N} (x_k - \hat{A}x_{k-1} - \hat{B}u_k)(x_k - \hat{A}x_{k-1} - \hat{B}u_k)^T .$$

IV. Conclusion

The above algorithms allow one to produced a stable estimation of the matrixes of parameters and covariances of the perturbation vectors in uncertain dynamic systems and thereby increase the accuracy of the adaptive control system.

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