



AUTOMATION OF THE CALCULATION OF UNCERTAINTY MEASUREMENTS

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Abstract: The work presented in this report described a program that allows calculating the results of direct and indirect measurements, evaluates their standard, total and extended type A and B uncertainties by reduction and linearization methods, calculate the sensitivity coefficients of the output (measured) value estimation, to changes in the estimates of input quantities (BB), the contribution of standard uncertainties (CH) BB to the total CH of type A and type B, determine the coverage factor, to make the budget of uncertainty of measurements.

Keywords: program, measurement, direct, indirect, measurement, standard, total, expanded uncertainty, type A, type B, method, reduction, linearization, coefficient, sensitivity, coverage factor, budget.

Introduction

According to the compulsory requirements of documents of international organizations, in particular, the International Organization for Standardization (ISO), International Bureau of Weights and Measures, International Electrotechnical Commission (IEC), International Federation of Clinical Chemistry (IFCC), International Union of Pure and Applied Chemistry (IUPAC), International Union of Pure and Applied Physics (IUPAP), International Organization of Legal Metrology (OIML) - "Guide to the Expression of Uncertainty in Measurement" The GUM Manual [1], and the international standard ISO/IEC 17025 "General requirements for the competence of testing and calibration laboratories" [2] and the identical state standard of Uzbekistan O'z DSt ISO/ IEC 17025 [3], and a number and in the EURACHEM standards [4], a calibration or testing laboratory shall provide quantitative

measurement results with measurement uncertainty values (paragraphs 5.4.6.1 and 5.10.4.1).

At the WCIS-2012 conference, we reported the results of scientific research in this direction, in particular "Methodology for estimating the uncertainty of measurements of a number of acoustic quantities" [5] and "Uncertainties in the results of measurements of thermophysical properties of textile materials" [6].

1. Statement of a problem

It should be noted that calculations of measurement uncertainty require special knowledge in the field of mathematical statistics and the ability to work with statistical packages. As a rule, these requirements do not always correspond to the level of personnel of testing laboratories.

The solution of this situation is the automation of calculations, which leads not only to the acceleration of the work of laboratory personnel, but also to a reduction in the period of its training in estimating uncertainty.

The simplest implementation of the basic algorithm for estimating uncertainty are programs developed in the Excel [7].

2. The concept of the problem decision

When using the software, it is initially assumed that the mathematical measurement model (MMI), sources of uncertainties and their parameters are known. The algorithm for estimating the uncertainty of the measurement results is shown in Fig. 1.

MMI is a function of the relationship between the input (BB) and output (B_{yx}B) values

$$Y = f(X_j) = f(X_1, X_2, \dots, X_m), \quad (1)$$

where, Y – the desired (measured, output) value; f – a certain functional relationship, between B_{yx}B and BB; X_j – j- th input variable (j=1, 2,...,m); j – serial numbers of BB; m – quantity of BB (m=1, 2, ..., m)

3. Realization of the concept

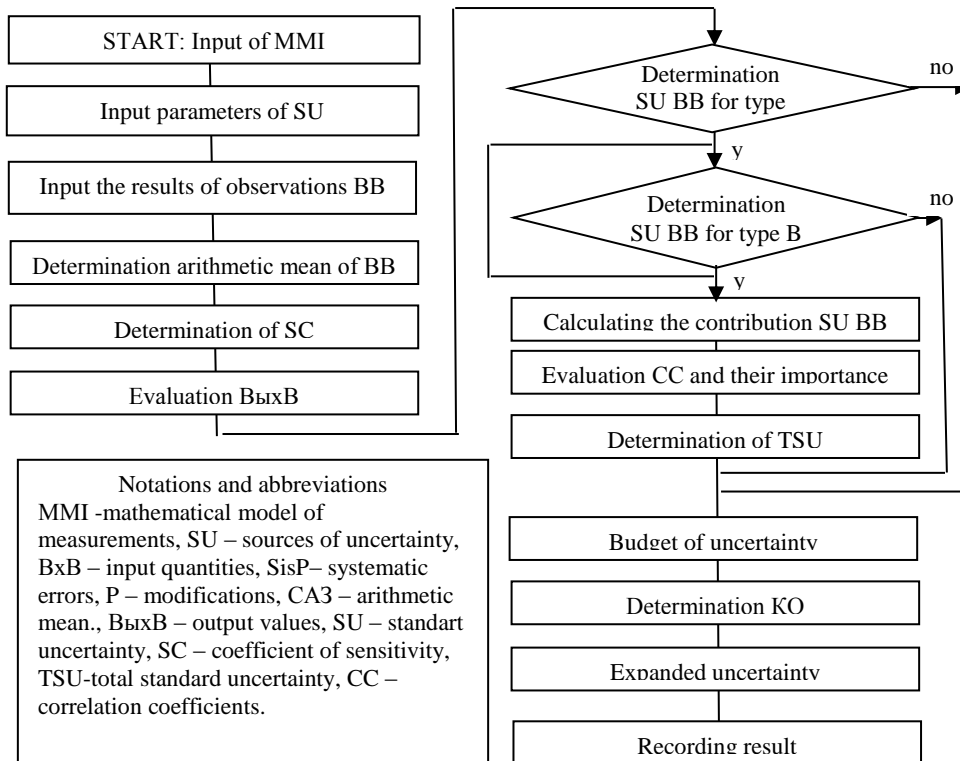


Fig. 1. Algorithm for estimating the uncertainty of measurement results.

The exponent BB (cell G6: G8) is equal to one or minus one.

In cells C11: F13, the parameters of sources of uncertainties (SU) are introduced, which are the values of the non-excluded systematic errors δX_i due to the calculation with limited accuracy (rounding the calculation result, the last digit of the number), the scale value of the measuring instrument (SI) a reading device, a variation of the readings of SI, etc., which are estimated by type B, assuming a uniform law of distribution of their probabilities. The values of the results of observations x_j BB X_j are collected into cells C4: E8.

Sequence of work with the program. Depending on the model of the problem being solved (Table 1, cells F3, F4), the model code is written to cell G5, for example, 1, if the model F3 is selected, or 2 if model F4 is selected.

The model F3 is the product and / or the quotient BB X_j, and the model F4 is the sum or difference BB, where X₁, X₂, X₃ are the input quantities, α, β, γ are the exponents BB.

The mean of the results of the observation of the j-th explosive for n_j> 1 is estimated by the formula (2) and reflected in cells C16: C22.

$$\bar{x}_j = \frac{1}{n_j} \cdot \sum_{i=1}^{n_j} x_{ji} \quad (2)$$

Where, n_j – the number of observations in the measurement j-th BB.

In the case of multiple observations of explosives, an estimate of the BB is obtained using the linearization method (ML) from formula (3) (cell C26) or the reduction method (MP) by formula (4) (cells H4: H8)

$$y = \bar{y} = f(\bar{x}_j), \quad (3) \quad \text{wher, } Y = \bar{Y} - \text{evaluation ВыхВ, equal (with multiple observations) arithmetic mean.}$$

$$y = \bar{y} = \frac{1}{n} \cdot \sum_{i=1}^n y_i, \quad (4)$$

Table. 1.

Program for calculating measurement uncertainty

A	B	C	D	E	F	G	H
2		BB and their units			The measurement model and its code		Y, kPa
3	<i>i</i>	x₁, kN	x₂, mm	x₃, mm	$Y = X_1^\alpha \cdot X_2^\beta \cdot X_3^\gamma$	1	МП
4	1	219	248	120	$Y = X_1^\alpha + X_2^\beta + X_3^\gamma$	2	7358,9
5	2	218	251	122	1		7119,1
6	3	219	239	128	$\alpha =$	1	7158,7
7	4	218	251	118	$\beta =$	-1	7360,4
8	5	219	248	118	$\gamma =$	-1	7483,6
9	n	5	5	5	5	∞	5
10	Unexpected systematic measurement errors					P	Coefficient of units BB
11	δx_1	8,744	1	1	1	0,95	1000000
12	δx_2	0	0	0	0		
13	δx_3	0	0	0	0		
14	Budget uncertainty						
15	Input value, Xi	Evaluation of input value, xi	Type of uncertainty	CH, u(xi)	Number degrees of freedom, v_i	Coefficient of sensitivity, c_i=∂f/∂x_i	Contribution of uncertainty, u_i(y)
16	x1, kN	218,6	Тип А	0,2	4	33,4	8,2
17			Тип В	8,7	∞		291,8
18			СН	8,7			
19	x2, mm	247,4	Тип А	2,2	4	-29,5	-65,0
20			Тип В	0,29	∞		-8,5
21			СН	2,2			
22	x3, mm	121,2	Тип А	1,9	4	-60,2	-111,7
23			Тип В	0,29	∞		-17,4
24			СН	1,9			
25	Measured (output) value, Y	Measurement result, y	A type uncertainties	Total SU, u(y)	Effective number of degrees freedom v_{eff}	Coefficient of coverage, k	Expanded uncertainty, U
26	Y, kPa методом приведен ия	7296	Тип А	129,5	149	1,96	627
27			Тип В	292,5			
28			ССН	319,9			

The number of observations BB can be different, in particular:

- а) $n_1 = n_2 = \dots = n_m$; б) $n_1 = 1, n_2 = n_3 = \dots = n_m \neq 1$;
- в) $n_1 = n_2 = \dots = n_{m-1} = 1, n_m > 1$.

If, BB in cell C4:E8 are introduced in multiples and international units SI, then to get the value ВыхВ in units SI in cell H11 the corresponding

conversion factor is introduced, for example, in the case under consideration – 1000000.

Standard uncertainties $u_{A_j}(x_j)$ for type A. The program defines SU $u_{A_j}(x_j)$ type A only if the number of observations n_j for the measurement of the j -th input value $n_j \geq 3$ (C9:E9). Estimation of SU by type A is carried out by the formula

$$u_{A_j}(x_j) = S(\bar{x}_j) = \sqrt{\frac{D(x_j)}{n_j}} = \sqrt{\frac{1}{n_j(n_j-1)} \sum_{i=1}^{n_j} (x_{ji} - \bar{x})^2} \quad (5)$$

where, $u_{A_j}(x_j)$ – SU type A $u_{A_j}(x_j)$,

$S(\bar{x}_j)$ – estimation of standard deviation the results of observations j -th BB;

Standard uncertainties $u_{B_j}(x_j)$ of type B $u_{B_j}(x_j)$ derived from following formula

$$u_{B_j}(x_j) = \sqrt{\sum_{N=1}^m u_{BN}^2(x_j)} \quad (6)$$

$$u_{BN}(x_j) = \frac{a_j}{\alpha_j} \quad (7)$$

where $u_{BN}(x_j)$ - N -th constituent SU of type B $u_{BN}(x_j)$; $a_j = \delta x / 2$ – half-width of the probability distribution interval of the measurement result j -th BB; δx – accuracy of calculation (rounding the result of the calculation), the last digit of the number, the scale value of the scale of the measuring instrument (MI), resolving device resolution MI, variation of readings MI; α_j – coefficient corresponding to the distribution law for the given j -th input quantity (normal $\alpha_j = 2$ for confidence level (probability) $P=0,95$, uniform $\alpha_j = \sqrt{3}$, triangular $\alpha_j = \sqrt{6}$), for the arc sine law $\alpha_j = \sqrt{2}$ and etc.; $N = 1, 2, \dots, m$ – serial numbers of SU of type B of input values x_j .

The values of the SU input values of type B in the program are reflected in the cells E17, E20 and E23.

The coefficients of sensitivity are calculating

$$u_c(y) = \sqrt{\sum_{j=1}^m u_j^2(y) + 2 \sum_{j=1}^{m-1} \sum_{L=j+1}^m u_j(y) \cdot u_L(y) \cdot r(\bar{x}_j, \bar{x}_L)} \quad (12)$$

where $u_j(y) = \sqrt{u_{A_j}^2(y) + u_{B_j}^2(y)}$ – j -th

(SC) $\partial f / \partial x_j, \partial f / \partial x_L$ estimates of the $u_{A_j}(x_j)$ and $u_{B_j}(x_j)$ changes in the estimates of BB x_j and x_L , respectively (G16:G22).

Calculation of the contribution of SU in TSU of type A and type B (H16: H23) is carried out according to formulas

$$u_{A_j}(y) = \frac{\partial f}{\partial x_j} \cdot u_{A_j}(x_j); u_{A_j}(y) = \frac{\partial f}{\partial x_L} \cdot u_{A_j}(x_L), \quad (8)$$

$$u_{B_j}(y) = \frac{\partial f}{\partial x_j} \cdot u_{B_j}(x_j); u_{B_j}(y) = \frac{\partial f}{\partial x_L} \cdot u_{B_j}(x_L), \quad (9)$$

where $u_{A_j}(y)$ – j -th contribution SU BB of type A in TSU, $u_{B_j}(y)$ – j -th contribution SU BB of type B in TSU.

The correlation coefficients (CC) between BB x_j and x_L ($L \neq j$) are determined by formula (10), and their significance is estimated using the Student's criteria according to the formula according to which the CC is significant if

$$r(\bar{x}_j, \bar{x}_L) = \frac{1}{n(n-1)} \cdot \sum_{i=1}^n (x_{ji} - \bar{x}_j) \cdot (x_{Li} - \bar{x}_L) / (u(\bar{x}_j) \cdot u(\bar{x}_L)) \quad (10)$$

$$\left| \frac{r(\bar{x}_j, \bar{x}_L) \cdot \sqrt{n-2}}{\sqrt{1-r^2(\bar{x}_j, \bar{x}_L)}} \right| > t_{p, (n-2)} \quad (11)$$

where $t_{p, (n-2)}$ – the value of the coefficient (quantile) for the results of observations of explosives (random variables) having a Student's distribution with $(n-2)$ degrees of freedom;

$u(\bar{x}_j)$ - SU BB x_j ;

$u(\bar{x}_L)$ - SU BB x_L .

After determining all components of uncertainty measurement, their total standard uncertainty $u_c(y)$ is estimated in accordance with the law of propagation of uncertainty [1] (cells E26: E28).

contribution SU BB in STU.

Further, the expanded uncertainty of the measurement result (cell H26) is calculated as

follow

$$U = k \cdot u_c(y), \quad (13)$$

where k – coefficient of coverage, defined as the Student's coefficient for the effective number of degrees of freedom ν_{eff} , calculated according to the formula of Welch-Suttswait:

$$\nu_{eff} = (n-1) \left(1 + \frac{u_B^2(y)}{u_A^2(y)} \right)^2 \quad (14)$$

The result of the measurement is written in the form

$$Y = \bar{y} \pm U, p = 0,95. \quad (15)$$

The intermediate results obtained during the implementation of the basic algorithm are conveniently presented in the form of an uncertainty budget, which includes a list of all BBs, their estimates together with the standard measurement uncertainties assigned to them, the sensitivity coefficients, the number of degrees of freedom, the measurement result, the total standard uncertainty, the effective number of degrees' freedom, coverage ratio and increased uncertainty (A14: G28).

Conclusion

Consequently, the proposed program allows to calculate the results of measurement of direct, indirect, joint and cumulative measurements, estimate their standard, total and expanded uncertainties of type A and B, by methods of reduction and linearization, calculate the sensitivity coefficients of the estimation of the ВыхВ in the

estimates of BB, type A and type B, to determine the coverage factor, to compile an uncertainty budget, in cases where:

- the measurement model is a product, a quotient, a sum and / or a BB difference;
- the number of observations for the measurement of all input quantities are the same (equal) and / or different.

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