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SUSTAINABLE ALGORITHMS FOR SYNTHESIS OF LOCAL-OPTIMAL ADAPTIVE DYNAMIC OBJECT MANAGEMENT SYSTEMS

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Abstract: *Stable algorithms for the formation of control actions in local-optimal adaptive control systems for dynamic objects are given. Considering that the initial equations for estimating the parameters of an object and a control device, as a rule, are ill-conditioned, it becomes necessary to use regular methods. The stable algorithms for finding the desired solutions based on the methods of the minimum pseudoinverse matrix and singular decomposition, which contribute to improving the accuracy of the formation of control actions, are given.*

Key words: *closed-loop control system, local-optimal adaptive control system, control action, robust algorithms, minimal pseudoinverse matrix, singular decomposition.*

I. Introduction

The main problem of parametric optimization consists in determining the parameters of the control algorithm from the condition of minimizing the chosen quality criterion [1-5]. Complex and time-consuming is the solution of the problem of parametric optimization for multimode objects or objects with slowly varying parameters when the rate of change of the parameters of the object is small and on the optimization interval they can be considered constant. The problem of parametric optimization can be reconciled with the general problem of synthesis of an adaptive control system [2,4,6]. In this case, both theoretical methods and numerical procedures are used to solve the problem of parametric optimization.

In the theoretical approach, adaptive control algorithms are defined, in which the parameters are functions of the coefficients of the mathematical model of the control object or depend on their specific relationships [4-7].

When using the second approach to the solution of the problem of parametric optimization, the control algorithm is known, and by modeling, the necessary ranges of the parameters of the control algorithm are determined, and on the basis of these results, functions for the adjustment of the parameters of the control algorithm are constructed. However, only a suboptimal solution can be obtained here [6-8].

Recursion algorithms [9-11] are most often used to estimate the coefficients of equations from observable data, allowing identification in the normal operation mode of the object. The control of the object leads to the degeneration of the information matrix and thereby prevents the identification of the object or the required optimal control law, determined by the identifiable parameters of the object.

There are various approaches to solving this problem. To prevent unidentifiability, various methods were proposed [7,10]: the addition of noise to the controller, the inclusion of several controllers in the control system and their connection by some algorithm, and others.

In [7,12] the general form of control as a function of unknown parameters of a linear object is given, for which the problem of identifiability is removed in the sense that the nonidentifiability of the parameters of the object does not entail the unidentifiability of the required control. It is shown [7] that locally optimal control belongs to this species. In this connection, the identifiability of the optimal control law takes place in locally optimal control systems.

II. Formulation of the problem

Very often, when synthesizing a regulator in closed systems, methods of locally optimal adaptive control are used [5, 8, 12, 13]. Consider a linear control object described by equation

$$x_{t+1} = Ax_t + Bu_t + w_{t+1}, \qquad (1)$$

where $x_t \in R^n$ – is the measured state, $\{w_t\}$ – is uncontrolled independent random perturbations satisfying condition

$$Ew_t = 0,\ Ew_t w_t^T = Q,$$

A and B are unknown matrices of dimension $n \times n$ and $n \times m$.

We take the control law in the form

$$u_t = \theta^T(\Omega_t)x_t,$$

where $\theta(\Omega)$ is the given matrix function, and Ω_t is the current estimate of the matrix $\Omega^T = [A \vdots B]$, obtained from the relations

$$\psi_{t+1} = \Omega^T \Phi_t + w_{t+1},$$

where Ω is an unknown matrix of dimension $n_\Phi \times n_\psi$; $\Phi_t \in R^{n_\Phi}$ and $\psi_t \in R^{n_\psi}$ are the vectors available to the measurement.

We define the sequence of estimations of matrices Ω according to the method of least squares on the basis of the recurrence relation [12]:

$$\Omega_{t+1} = \Omega_t + \Gamma_{t+1}^{-1}\Phi_t\left(\psi_{t+1} - \Omega_t^T\Phi_t\right)^T,$$

where Ω_0 is an arbitrary matrix of dimension $n_\Phi \times n_\psi$, and the matrix Γ_{t+1} has the form

$$\Gamma_{t+1} = \Gamma_t + \Phi_t\Phi_t^T,\quad \Gamma_0 > 0,$$

$$\Phi_t^T = \left(x_t^T, u_t^T\right), \psi_{t+1} = x_{t+1}.$$

Then the limiting law of control will be determined by the expression [7,9]

$$u_t = \theta^T(A,B)x_t,$$

$$\theta^T(A,B) = (H^T B)^{-1} H^T (S^T - A). \qquad (2)$$

The control law (2) minimizes the conditional mathematical expectation of the value along the trajectory (1) of the objective function

$$V(x_{t+1}) = (x_{t+1} - x_{t+1}^\theta)^T C (x_{t+1} - x_{t+1}^\theta),$$

while the nonnegative definite matrix $C = C^T$ satisfies the equality $H = CB$, and x_{t+1}^θ – is the state value of the reference trajectory determined by equation $x_{t+1}^\theta = S^T x_t$, and at $S = 0$ control (2) will coincide with locally optimal control.

Also consider the control object specified in the form

$$A(z^{-1})y_{t+1} = B(z^{-1})u_t + w_{t+1}, \qquad (3)$$

where $y_t \in R^l$ - measured outputs, $u_t \in R^m$ - control.

We introduce the following notation

$$\Omega^T = (-A^{(n)}, ..., -A^{(1)}, B^{(n-1)}, ..., B^{(0)}),$$

$$\Phi_t^T = (y_{t-n+1}^T, ..., y_t^T,\ u_{t-n+1}^T, ..., u_t^T),$$

where $\psi_{t+1} = y_{t+1}$.

Then, taking into account that

$$u_t = \theta^T(\Omega_t)\eta_t,$$

$$\eta_t^T = (y_{t-n+1}^T, ..., y_t^T,\ u_{t-n+1}^T, ..., u_t^T),$$

the locally optimal control law takes the form:

$$u_t = (H^T B^{(0)})^{-1}[(A^{(1)} + S^{(1)})y_t + ...$$
$$+ (A^{(n)} + S^{(n)})y_{t-n+1} + (-B^{(1)} + D^{(1)})u_{t-1} + ... \qquad (4)$$
$$+ (-B^{(n-1)} + D^{(n-1)})u_{t-n+1}],$$

where $S^{(i)}, i = \overline{1,n}$ and $D^{(j)}, j = \overline{1,n-1}$ are arbitrary matrices of dimension $l \times l$ and $l \times m$ respectively, H – is the matrix $m \times l$ such that $\det H^T B^{(0)} \neq 0$.

III. Solution of the task

Thus, for the objects described by equations (1) and (3), the locally optimal control law is formed on the basis of expressions (2) and (4), respectively. In expressions (2) and (4), square matrices of the form $G = H^T B$ and $G^{(0)} = H^T B^{(0)}$ are reversed. These matrices can be ill-conditioned, which ultimately leads to the need to construct regular algorithms for the formation of the desired estimates.

In addition, in practical problems, elements, for example, matrices G, are often known to us approximately. In these cases, instead of matrix G, we are dealing with some other matrix $\tilde{G}$ such that $\|\tilde{G}-G\| \le h$, where the meaning of the norms is usually determined by the nature of the problem. Having a matrix $\tilde{G}$ instead of matrix G, we can not make a definite judgment about the degeneracy or nondegeneracy of the matrix G. But there are infinitely many such matrices $\tilde{G}$, and within the limits of the level of error known to us they are indistinguishable. Among such "possible exact systems" there may be degenerate ones.

To impart numerical stability to the inversion procedure for matrices G and $G^{(0)}$, it is advisable to use the concepts of regular and stable estimation methods [14–16]. Below is the algorithm for estimating the inverse matrix in equation (2), taking into account its possible degeneracy. The same algorithm can also be used to estimate the inverse matrix in equation (4).

It is known [17,18] that the problem of calculating G^{+} is equivalent to solving an extremal problem: find $Z \in U^{*}$ such that

$$\|GZ-I\|_{m\times m} = \inf\{\|GZ-I\|_{m\times m} : Z \in U^{*}\}. \quad (5)$$

Solution Z of problem (5) is unique and coincides with G^{+}. This allows us to construct an algorithm for finding a stable approximation to G^{+} for a given $\tilde{G}$ using Tikhonov's regularization method for extremal problems [15] with the choice of the regularization parameter according to the generalized residual principle.

We introduce the smoothing functional Tikhonov

$$M_h^{\alpha}[Z] = J_h^2(Z) + \alpha\|Z\|_*^2 = \\ = \|\tilde{G}Z - I\|_{m\times m}^2 + \alpha\|Z\|_*^2, \ \alpha > 0, \ Z \in U^{*}. \quad (6)$$

As is known [14, 15], the minimization problem for the quadratic functional (6) in the Euclidean space U^{*} has a unique solution $\tilde{Z}_{\alpha}$. Therefore, at $\alpha > 0$, we can define a generalized residual

$$\rho_{\xi}(\alpha) = J_h(\tilde{Z}_{\alpha}) - Ch\|\tilde{Z}_{\alpha}\|_* - \mu_h^{\beta},$$

$$C = const \ge 1, \ \xi \equiv (h, \beta).$$

The value μ_h^{β} represents a β approximation to the modified incompatibility measure μ_h [19]:

$$\mu_h \equiv \inf\{ J_h(Z) + h\|Z\|_* : Z \in U^{*}\} = \\ = \inf\{ \|\tilde{G}Z - I\|_{m\times m} + h\|Z\|_* : Z \in U^{*}\},$$

i.e. $0 \le \mu_h^{\beta} - \mu_h \le \beta$.

According to [15], equation $\rho_{\xi}(\alpha) = 0$ has, under condition $\|I\|_{m\times m} > \mu_h^{\beta}$, a unique solution $\alpha(\xi) > 0$. As an approximation to the pseudo-inverse matrix G^{+}, we can take the extremal $\tilde{Z}_{\alpha(\xi)}$ of functional (6) corresponding to parameter $\alpha = \alpha(\xi)$. Then $\tilde{Z}_{\alpha(\xi)} \to G^{+}$ at $\xi = (h, \beta) \to 0$.

A very effective approach for calculating the matrix G_h^{+} is also the algorithm [20], based on the method of the minimum pseudoinverse matrix and consisting in determining the numbers $\hat{\rho}_1, ..., \hat{\rho}_m$, which is $\hat{\rho}_1 \ge ... \ge \hat{\rho}_m \ge 0$ and

$$\sum_{k=1}^{m} \theta(\hat{\rho}_k^2) = \inf\left\{ \sum_{k=1}^{m} \theta(\rho_k^2) : \rho_1 \ge ... \ge \rho_m \ge 0; \right. \\ \left. \sum_{k=1}^{m} (\rho_k - \rho_k^h)^2 = h^2 \right\},$$

where ρ_k^h, $k = 1,2,...,m$ – given singular numbers of the matrix G_h.

Following [20], one can show the validity of the following relations:

$$\|\hat{G}_h - G_h\|^2 = \|U_h \hat{D}_h V_h^T - U_h D_h V_h^T\|^2 = \\ = \|\hat{D}_h - D_h\|^2 = \sum_{k=1}^{m} (\hat{\rho}_k - \rho_k^h)^2 = h^2,$$

$$\left\|\hat{G}_h^+\right\|^2 = \left\|V_h \hat{D}_h^+ U_h^T\right\|^2 = \left\|\hat{D}_h^+\right\|^2 \le \left\|G^+\right\|^2, \quad \forall G \in U_h,$$

where $\hat{G}_h \equiv U_h \hat{D}_h V_h^T$ – singular value decomposition for the matrix G_h, U_h and V_h are orthogonal matrices,

$$D_h \equiv diag(\rho_1,...,\rho_m) \in U .$$

Then the minimum pseudo-inverse matrix can be calculated based on the expression

$$\hat{G}_h^+ = V_h \hat{D}_h^+ U_h^T ,$$

where

$$\hat{D}_h^+ = diag[\theta(\hat{\rho}_1),...,\theta(\hat{\rho}_m)] \in U^* .$$

The given algorithms allow stabilizing the procedure of forming and developing control actions in locally optimal adaptive control systems for dynamic objects and improving the quality indicators of control processes.

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