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FUZZY-MULTIPLE TASKS OF MULTICRITERIAL OPTIMIZATION IN RISK CONDITIONS

Хатарлар шароитида оптималлаштиришнинг ноаниқ кўп мезонли масалаларини ечиш ва оптималлаштириш масалаларини ечишга бўлган ёндашувлар кўриб чиқилган. ноаниқ шароитларда хатарларни баҳолаш учун қарор қабул қилишнинг стандарт масалаларидаги чекланишлар тизимини бўлиши мумкин бўлган йўқотишлар тўплами билан тўлдириш, хусусан, танланган сценарийлар учун уларнинг оқибатлари (йўқотишлари) моделини бошқарилувчи параметрларнинг функциялари каби қуриш ҳамда ҳар бир сценарий учун йўқотишларга нисбатан мумкин бўлган даражадаги экспертли чекланишларни қўйиш таклиф этилган.

Таянч сўзлар: *кўп мезонли оптималлаштириш, хатар, ноаниқ тўпламлар назарияси, тегишлилик функциялари, нокррект масала, нейрон тармоқлари, асалари галаси алгоритми.*

Рассматривается подходы к решению оптимизационной задачи и решение нечеткой многокритериальной задачи оптимизации в условиях риска. для оценки рисков в нечетких условиях предлагается дополнить систему ограничений стандартной задачи принятия решений набором ограничений по возможным потерям, а именно, для избранных сценариев построить модель их последствий (ущербов) как функций управляющих параметров и накладывать экспертные ограничения по приемлемому уровню относительного ущерба для каждого сценария.

Ключевые слова. *Многокритериальная оптимизация, риск, теория нечетких множеств, функция принадлежности, некорректная задача, нейронные сети, алгоритм пчелиного роя.*

Approaches to solving the optimization problem and solving a fuzzy multicriteria optimization problem under risk conditions are considered. To assess risks in fuzzy conditions, it is proposed to supplement the system of constraints of a standard decision-making task with a set of restrictions on possible losses, namely, for selected scenarios, to build a model of their consequences (damages) as functions of control parameters and impose expert limitations on an acceptable level of relative damage for each scenario.

Keywords. *Multi-criteria optimization, risk, fuzzy set theory, membership function, incorrect problem, neural networks, bee swarm algorithm.*

Introduction

Efficiency of functioning of fairly complex real objects or processes, as a rule, is characterized by a set of partial criteria, often in mutual contradiction with each other, when improvement in one of the indicators leads to deterioration in the other and vice versa, and meeting the requirements of all criteria is impossible. In addition, criteria, as well as limitations, are usually formulated very imprecisely. Under these conditions, finding effective solutions is impossible without taking into account inaccurate, high-quality information about the preferences of various criteria, about the desired nature of the processes - the growth or reduction of quality parameters, about the range of their changes.

1. Distinguishing characteristics, criteria and models of tasks.

As the task becomes more complex, the role of such inaccurate qualitative information increases and in many cases becomes decisive [1]. As indicated in [2], if there are only two criteria, there are inevitably subjective factors in optimization problems that are associated, for example, with the ranking of particular criteria. To a certain extent, such difficulties can be eliminated by simplifying the formulation of the problem. For example, you can select any one main criterion of quality, and the rest to consider as limitations. Another way is to use the method of successive assignments [2]. However, such approaches lead to the coarsening of the initial task and do not eliminate the qualitative, subjective elements, transferring them from the statement of the problem to the stage of analyzing the results. The need for quantitative ranking of particular criteria and uncertainty when describing them in multi-objective optimization tasks are objectively sources of subjectivity, uncertainty. The need to use qualitative information is recognized by many researchers, and various ways of formalizing and solving this problem have been proposed. To describe the particular criteria and limitations, they were offered the use of desirability functions. The latter take values that continuously increase from 0 to 1 when the corresponding quality parameters change from least to the most desirable values. The specific type of desirability functions is given by the decision maker (DM), based on his subjective perceptions. By convolving the particular functions of desirability, a global criterion of the quality of the process is built, the maximization of which delivers the optimum. This method is widely used in planning experiments when searching for optimal conditions [3]. It has been successfully used in solving problems of optimizing processes of chemical technology, material processing, in metallurgy and other industries [4-6]. From the definition of the functions of desirability it follows that in solving optimization problems, both in form and in their semantic content are actually equivalent to the membership functions of fuzzy sets. Another approach to the methods of formalizing the description of fuzzy, qualitative characteristics was proposed by L.A Zadeh [7].

The theory of fuzzy sets, especially its conceptual basis and mathematical apparatus for working with objects of a linguistic nature, proved to be fruitful, effective means of posing and solving problems of multicriteria optimization in the presence of non-statistical uncertainties. It should be noted that there is an extremely large variety of such problems, and therefore there is no single universal method for solving them [8]. The main results, achievements and problems in the field of fuzzy multicriteria optimization and decision making are described in the review literature [9–11] and production character [12–14]. In [15-18], in order to build decision-making models under uncertainty conditions, a linguistic approach is used, which allows formalizing the problem in the presence of criteria and constraints described in natural language. In [19-22], the tasks of fuzzy multicriteria optimization are solved in the presence of fuzzy coefficients of the relative importance of the criteria.

In the papers [22, 23] proposed an approach based on the theory of possibilities developed by L.A Zadeh on the basis of the theory of fuzzy sets, in [24] the problems of multi-criteria decision-making in the presence of uncertainties of both fuzzy and probabilistic types are considered.

We formulate the main features of multi-criteria tasks in the presence of ill-defined criteria.

Currently, the majority of researchers note that the key features of the formulation of these problems are [8], a) the existence of many alternatives; b) there are many limitations that must be considered when choosing alternative solutions; c) the existence (explicitly or implicitly) of the preference function that matches each alternative with the gain (or loss) that will be obtained by choosing this alternative.

A specific feature of fuzzy problems is also symmetry between goals and constraints, which eliminates the differences between them in terms of their contribution to the formulation and solution of problems. We formulate this position in a constructive form.

Constructive, fully reflecting the qualitative character of the assignment of preferences in a multicriteria task is the approach proposed by R. Yager, based on a generalization of the concepts of concentration and tension [22].

In this case, one of the most important problems is the formation of a global criterion. The convolution procedure cannot be formalized to the end and is determined by the specifics of the problem, goals, experience and intuition of the researcher. In [6], it was shown that different methods of convolution of criteria can lead to significantly different final results, which indicates the decisive importance of the stage of the formation of a global criterion when solving multicriteria problems. Therefore, despite the absence of a general orientation, it is advisable to consider some key points in the process of forming convolution of particular criteria, to conduct a comparative analysis of the most frequently used methods for constructing a generalized quality indicator when describing particular criteria by membership functions.

As noted earlier, when solving problems of multi-criteria evaluation and optimization, it is necessary to take into account the unevenness of particular quality criteria. In the case of a large number of criteria, the task of directly determining the ranks of criteria turns out to be very difficult and even insoluble for experts due to the limitations of the psycho-physiological capabilities of a person. Moreover, in the case of a comparison of two alternatives, the expert is usually able to adequately determine which of the considered characteristics (importance) is more pronounced, and also qualitatively (verbally) assess how big the difference is between the characteristics observed by the two alternatives.

When formulating a multi-criteria optimization problem, the condition of mandatory satisfaction of all particular criteria and constraints, i.e. at. optimum point, all desirability functions must be non-zero. It also requires that the optimum criteria be satisfied as much as possible. In other words, it is considered undesirable that the value of the generalized criterion increases with the improvement of a number of quality indicators due to the deterioration of the others. In the terminology of the theory of decision making, the last requirement is equivalent to the condition that the optimum point belongs to the Pareto set [8].

There are a large number of publications on decision making at risk. The minimax approach, optimization of the expected utility, minimization of the average damage or the probability of an adverse event, models of stochastic programming, etc. are widely used.

However, these statements are not enough to make decisions in fuzzy conditions, where it is impossible to focus on the average efficiency of decisions, since they are justified in the case of repeatedly repeated situations, while risky situations are unique, they can happen tomorrow and never happen. The latter are characterized by the possibility of extremely unlikely, but exceptionally large losses bordering on the survival of the system in question. It is clear that traditional risk indicators such as variance are inadequate in this case.

In this regard, in order to assess risks in fuzzy conditions, it is proposed to supplement the system of constraints of a standard decision task with a set of restrictions on possible losses, namely, for selected scenarios, to build a model of their consequences (damages) as functions of control parameters and to impose expert restrictions on an acceptable level of relative damage for each script.

Obviously, to find the ideal option “maximum yield - minimum risk” is possible only in very rare cases. Therefore, the following approaches are proposed for solving this optimization problem (Table 1).

Table 1

Approaches to solving the optimization task

№	*Approaches*	*Model*
1	*The “maximum gain” approach is that the one that gives the highest result (maximum F) is selected from all the options with acceptable risk for the decision maker* ***(*** $R_{ad.m}$ ***).***	$\mathrm{F} \to \max$, $R = R_{пр.доп}$, $\sum_j x_j K_j \subset K$.
2	*“Optimal probability” approach is that of the possible solutions is chosen the one in which the probability of the result is acceptable for the decision maker, where* $M(F)$ *– expected F.*	$M(F) \to \max$, $\sum_j x_j K_j \subset K$.
3	*The combination of approaches “optimal probability” and “optimal variability”. The variability of indicators is expressed by their standard deviation and coefficient of variation, where CV (F) is the coefficient of variation F.*	$CV(F) \to \min$, $\sum_j x_j K_j \subset K$.
4	*“Minimum risk” approach. Of all the possible options, the one that allows you to get the expected gain, i.e. maximum permissible value of F with minimal risk.*	$F = F_{пр.доп}$, $R \to \min$, $\sum_j x_j K_j \subset K$.
5	*“Maximum Yield - Minimum Risk” Approach*	$\mathrm{F} \to \max$, $R \to \min$, $\sum_j x_j K_j \subset K$.

In multi-criteria tasks it is difficult to evaluate the solution of the problem in the complex of all criteria. The most common are additive convolution method and estimation by a decision maker (DM) [7-9].

2. Stages of the formation of tasks

It is proposed to use fuzzy methods to evaluate alternative solutions. The solution of a multi-criteria optimization problem can be one of the following steps [8-11]:

- Formation of the objective function in a fuzzy setting.
- Definition of evaluation criteria values in fuzzy form.
- Development of membership functions for criteria.
- Defining a rule base and / or preference base for criteria.
- Calculate target function values.
- Defasification (reduction to a clear view) of the objective function.

3. S - parametric programming problem

Denote by $F(X,\Lambda)$ operation of convolution of partial optimality criteria, where $\Lambda \in D_\Lambda \subset R^n$ - weight factor vector

$(\Lambda = \{\lambda_i\}, i = \overline{1,s}$); $D_\Lambda = \{\lambda_i | \lambda_i \geq 0, \sum \lambda_i = 1, i \in [1:s]\} y \in R^m, y \leq g$ - the set of valid values of this vector.

Parametric programming problem with S independent parameters $\Lambda = \{\lambda_i\}, i = \overline{1,s}$, or *S*-parametric programming problem in the matrix form is written as follows:

$$F(X,\Lambda) = (\overline{a}_0 + \Lambda' \overline{b})X + \overline{e}\Lambda \to extr,$$

$$\sum_j XK_j \subset K,$$

$$\lambda \in R^s.$$

Here $X = \{x_j\}, j = \overline{1,n}$ – solving the S-problem of parametric programming, *K={y| }* – given convex subset of space R^m, $\overline{a}_0, \overline{b}, \overline{e}$ - coefficients that are fuzzy values, usually represented as fuzzy sets with given functions of belonging $\mu_{\overline{a}_0}(a_0)$ $(\overline{a}_0 \subset A_0)$, $\mu_{\overline{b}}(b)$ $(\overline{b} \subset B)$ and $\mu_{\overline{e}}(e)$ $(\overline{e} \subset E)$. $\sum_j XK_j$ understood as follows:

$$\sum_j XK_j = \{y \Big| y \in R^m, y = \sum_{i=1}^{n} x_i a_{ij},\ x_i \in X,$$

$$a_{ij} \in K_j \subset R^m, i = 1,...,n;\ j = 1,...,m\}$$

To solve the parametric programming problem with fuzzy source data (coefficients $\overline{a}_0, \overline{b}, \overline{e}$) three approaches are proposed:

1. By using various defuzzy operations on fuzzy sets $\overline{a}_0, \overline{b}, \overline{e}$ (integration, summation, averaging, etc.), you can get fuzzy estimates of the values of the coefficients a_0, *b, e* [8]. Then, entering them into the S-problem of parametric programming instead of fuzzy coefficients and writing the constraint in the form of the corresponding inequalities, we reduce the original problem to the form:

$$F(X,\Lambda) = (a_0 + \Lambda' b)X + e\Lambda \to extr(\min) \sum_j XK_j \leq g, \qquad \lambda \in R^s.$$

Note that due to the fuzziness of the description of the coefficients $\overline{a}_0$ and $\overline{b}$ evaluation of any decision $x(\lambda) \in X$ (and, accordingly, the values of the function $F(X,\Lambda)$ at $x=x(\lambda)$)is a fuzzy subset of the number axis of the base set *X*.

2. Reducing the solution of the original problem to solving linear programming problems for each discrete α - level [9].

As a result, fuzzy constraints take the following interval form:

$$P = \begin{cases} \sigma_\alpha(a_{i1})x_1 + \sigma_\alpha(a_{i2})x_2 + \ldots + \\ +\sigma_\alpha(a_{in})x_n \subseteq \sigma_\alpha(b_i), i = \overline{1,m}, \alpha = \overline{1,p}, \\ x_j \geq 0,\ j = \overline{1,n} \end{cases}$$

Here $X = \{x_j\},\ j = \overline{1,n}$ – solution of the multicriteria parametric programming problem at each discrete α -level, $\sigma_\alpha(a_{i,j})$ and $\sigma_\alpha(b_i)$ - interval values of coefficients $a_{i,j}$ and b_i on each discrete α -level.

3. Solution of the problem of multi-criteria optimization by an adaptive method [7]. Each iteration of these methods includes the analysis phase performed by the decision maker (DM) and the calculation phase performed by the multi-criteria optimization system.

Direct adaptive method for solving a multicriteria problem, which is studied in this paper, is based on the assumption of the existence of a "decision maker's function of preferences" $F(X,\Lambda) = (a_0 + \Lambda' b)X + e\Lambda$, which is defined on the set D_X allowable values of the vector of variable parameters X and performs the mapping of this set to the set of real numbers R. Moreover, the problem of multi-criteria optimization is reduced to the problem of choosing a vector $X^\bullet \in D_X$ ($X^* = \{x_j^*\},\ j = \overline{1,n}$) such that

$$\min_X F(X,\Lambda) = F(X^\bullet,\Lambda) \qquad X \in D_X .$$

It is assumed that upon presentation of the decision maker of the vector of parameters X, as well as the values of all partial optimality criteria $f_1(X), f_2(X), \ldots$, this person can evaluate the corresponding value of the preference function $F(X,\Lambda)$.

Let X the vector of variable parameters of the problem. The set of valid values for a vector is a bounded and closed set $D_X = \Pi \bigcap D$, where

$$\Pi = \{X \left| x_i^- \leq x_i \leq x_i^+, i \in [1:n]\right.\} \subset R^n$$

the set of permissible values of the parameters vector; D is a set of form bounding functions $g_i(X)$, sudh that

$$D = \{X \left| g_i(X) \geq 0,\ j = 1,2,\ldots\right.\} \subset R^n;$$

$R^n - n$ - dimensional space.

Vector optimality criterion $F(X,\Lambda) = (f_1(X), f_2(X), \ldots f_s(X), \lambda_1, \lambda_2, \ldots, \lambda_s)$ with values in space R^s defined in a set of valid values Π. The decision maker seeks to minimize on the set D_X each of the individual optimality criteria $f_1(X), f_2(X), \ldots f_s(X)$, which is conventionally written as

$$\min_I F(X,\Lambda) = F(X^\bullet,\Lambda), X \in D_X , \tag{1}$$

where $X^\bullet$- desired solution of a multicriteria problem. Partial criteria for optimality are assumed to be normalized (one way or another), so that $f_i(X) \in [0,1]\ \ i \in [1:s]$.

With each fixed vector $\Lambda \in D_\Lambda$ scalar convolution method reduces the solution of task (1) to solving a single-criterion task of global conditional optimization:

$$\min_X F(X,\Lambda) = F(X^\bullet,\Lambda) \qquad X \in D_X . \tag{2}$$

Note that in the case of additive convolution $F(X,\Lambda)$ $X^{\bullet}$ vector belongs to the set of Pareto efficient vectors [9].

This circumstance allows us to assume that in this case the function of preferences of the decision maker is not defined on the set D_X , and on the set D_A:

$$F:\Lambda \to R.$$

As a result, the multicriteria problem is reduced to the problem of choosing a vector $\Lambda^{\bullet} \in D_{\Lambda}$ such that

$$\min_{\Lambda} F(X,\Lambda) = F(X,\Lambda^{\bullet}) \qquad \Lambda \in D_{\Lambda} \tag{3}$$

Since usually $s \ll n$, the transition from task (1) to task (3) is important in terms of reducing computational costs.

The magnitude Ψ will be considered as linguistic variable with values from “Very, very bad” to “Excellent.” Fuzzy variable core Ψ denote by $\dot{\Psi}$ [8], so what is the value of the variable Ψ “Very very bad” matches $\dot{\Psi}=1$, and the value "Excellent" - $\dot{\Psi}=l$.

4. Stages of solving the problem by interactive iterative method

As a result, the multicriteria problem is reduced to the problem of finding the vector $\Lambda^{\bullet} \in D_{\Lambda}$, providing the maximum value of a discrete function $\dot{\Psi}(\Lambda)$:

$$\max_{\Lambda} \dot{\Psi}(\Lambda) = \dot{\Psi}(\Lambda^{\bullet}) = \dot{\Psi}^{\bullet} \qquad \Lambda \in D_{\Lambda} . \tag{4}$$

The general scheme of the considered method is iterative and consists of the following basic steps listed below [9].

Stage 1. Randomly generated sequentially s vectors $\lambda_1, \lambda_2, ..., \lambda_s$ and for each of these vectors the following actions are performed:

1) a multicriteria problem is solved

$$\min_{X} F(X,\Lambda) = F(X^{\bullet},\Lambda) \qquad X \in D_X \tag{5}$$

2) the decision maker presents the found solution $X^{\bullet}$, as well as the corresponding value of all particular optimality criteria $f_1(X^{\bullet}),\ f_2(X^{\bullet}),\ ...\ f_s(X^{\bullet})$;

3) the decision maker evaluates this data and enters into the task the corresponding value of its preference function $\dot{\Psi}(\Lambda_i)$.

Stage 2. Based on all available values. $\lambda_1, \lambda_2, ..., \lambda_s$ of vector Λ and the corresponding estimates of the function of preferences the following actions are performed:

1) function is built $\tilde{F}_1(X,\Lambda)$, approximating function $F(X,\Lambda)$ in the vicinity of points $\lambda_1, \lambda_2, ..., \lambda_s$;

2) the minimum of the function is found $\tilde{F}_1(X,\Lambda)$

$$\min_{A} \tilde{\Psi}_1(\Lambda) = \tilde{\Psi}(\Lambda_1^{\bullet}), \quad \Lambda \in D_{\Lambda};$$

3) with found vector $\Lambda_1^{\bullet}$ the problem of the form (5) is solved - the vector of parameters and the corresponding values of the partial optimality criteria are found, and then the decision maker is presented; The decision maker evaluates the specified data and enters into the system the corresponding value of its function of preferences $F(\Lambda_1^{\bullet})$.

Stage 3. Based on all values in the system $\lambda_1, \lambda_2, ..., \lambda_s$ of vector Λ and relevant preference function scores $F(X, \Lambda_1^{\bullet})$ function approximation is performed $F(X, \Lambda)$ in the vicinity of points $\Lambda_1, \Lambda_2, ..., \Lambda_k, \Lambda_1^{\bullet}$, $\tilde{F}_2(X, \Lambda)$ function is built according to the scheme of the first stage until the decision maker decides to stop the calculations.

The inputs of the fuzzy inference system are the weights of the partial optimality criteria - fuzzy terms $\lambda_{i,}$, $i = 1, 2, ...k \in [1:s]$. The output variable of the fuzzy inference system is a linguistic variable. Ψ, core of which $\overset{\bullet}{\Psi}$ takes values 1,2,…,*l*.

The relationship between input and output variables is described by fuzzy rules of the form

IF <values of input variables> THEN

<value of output variable> (ϕ_i) .

Here $\phi_i \in [0,1]$ - certainty coefficients (weights of fuzzy rules), which are equal to 1 in case of their uncertainty.

The set of values of these fuzzy input variables, output linguistic variables, and also the rules of fuzzy products form a fuzzy knowledge base.

5. Fuzzy-correct models of problems

The problem of constructing fuzzy models based on the conclusions of fuzzy rules in the face of uncertainty arises when assessing the state of poorly formalized processes. The advantage of fuzzy logic is the ability to use expert knowledge about this object in the form of statements (predicate rules): if - "inputs", then - "outputs". At the same time, it should be noted that the construction of fuzzy models of this type is often associated with the appearance of so-called fuzzy-incorrect problems.

Of particular note are the works of A. N. Tikhonov, V. Ya. Arsenin, and A. V. Yazenin in finding approximate solutions to ill-posed problems when constructing formalized evaluation models. However, the issues of solving fuzzy-incorrect problems are currently insufficiently studied.

Let the state of a weakly formalized process be described by specifying a sample of fuzzy experimental data (X_r, y_r), $r = \overline{1, M}$. Here $X_r = (x_{r1}, x_{r2}, ..., x_{rn})$ – the input n dimensional fuzzy vector, which is defined with its membership functions, and $y_r = \left(y_1, y_2, ..., y_M\right)$ –corresponding output vector.

It is required to construct fuzzy-correct models of decision-making tasks for assessing and predicting the state of a poorly formalized process, described in general terms by a set of fuzzy rules of productions (linguistic statements).

$$\bigcup_{p=1}^{k_i}\left(\bigcap_{j=1}^{n} x_j^i = a_{ij}^p - \text{с весом } w_{ip}\right) \rightarrow y_i^f = , \quad i = \overline{1, m} . \qquad (6)$$

$$= f(b_{i0}, b_{i1}, b_{i2}, ..., b_{in})$$

Here: a_{ij}^p – linguistic term by which the variable x_j^i is evaluated in row with the number *p* in the rules *i;*

w_{ip} – weight coefficient for rows *p* in the rules *i;*

$y_i^f = f(b_{i0}, b_{i1}, b_{i2}, ..., b_{in})$ – the output of the model (6) described by the rule *i*.

It is required to find such values of unknown coefficients b_{ij} *(* $i = \overline{1, m}$ *,* $j = \overline{1, n}$ *)* in the process of building a fuzzy model (6) that provide a minimum of the residual:

$$E = \sum_{r=1}^{M}\left(y_r - y_r^f\right) \rightarrow \min , \qquad (7)$$

where y_r^f – model output (6) corresponding to the input vector X_r .

The solution of this type of problem corresponds to the solution of the following equation:

$$Y = A \cdot B , \qquad (8)$$

where $A = \begin{bmatrix} \beta_{1,1},...,\beta_{1,m}, \quad x_{1,1}\cdot\beta_{1,1},...,x_{1,1}.\beta_{1,m}, \quad ..., \\ x_{1,n}\cdot\beta_{1,1},...,x_{1,n}\cdot\beta_{1,m} \\ \vdots \\ \beta_{M,1},...,\beta_{M,m}, \quad x_{M,1}\cdot\beta_{1,1},...,x_{M,1}\cdot\beta_{1,m},..., \\ x_{M,n}\cdot\beta_{M,1},...,x_{M,n}\cdot\beta_{M,m} \end{bmatrix}$ $\beta_{ir} = \frac{\mu_{d_i}(X_r)\cdot d_i}{\sum_{i=}^{m}\mu_{d_i}(X_r)}$,

$$d_i = \begin{cases} b_{i0} + b_{i1}x_1^r + b_{i2}x_2^r + ... + \\ + b_{in}x_n^r, \quad \textit{in case of linear dependence}, \\ b_{i0} + b_{i1}x_1^r + b_{i2}x_2^r + ... + b_{in}x_n^r + b_{in+1}\left(x_1^r\right)^2 + \\ + b_{in+2}\left(x_2^r\right)^2 + ... + b_{i2n}\left(x_n^r\right)^2, \\ \textit{in case of nonlinear} \\ \qquad \textit{dependence}, \\ term_i, \qquad \textit{in case of fuzzy terms}; \end{cases}$$

In this case, the construction of the desired fuzzy model is reduced to finding such a vector B, under which the condition $E = (Y - Y^f)^T \cdot (Y - Y^f) \to \min$.

In the proposed models, each input variable has its own membership functions $\mu(x,c,\sigma)$ fuzzy terms (for example, H - low, NA - below average, C - medium, Sun - above average, B - high), which are used in the equations.

In the process of developing a fuzzy risk assessment model based on the findings of fuzzy rules, one often encounters the problem of finding approximate solutions to fuzzy-incorrect problems. It should be noted that the methods designed to solve incorrect problems of decision support systems are developed only for a number of special cases of models (for example, for models based on classical logic). At the same time, there is currently no common approach to solving fuzzy-incorrect problems of this type.

To solve this problem, you can use the method of finding the neighborhood of the solution of ill-posed problems. To this end, we present some definitions and prove the assertions.

Definition 1. Primary information (about the object of study) is a fuzzy set with the membership function. **μ(x),** where **x∈X**.

Definition 2. Primary information is called fuzzy-compact if any level set, except for zero, is compact on the space X, i.e. $\forall\alpha\in(0,1],\ A_\alpha = \{x : \mu(x) \ge \alpha\}$— compact area on space.

Statement 1. Primary information, the membership function of which is shown in Table. 2, is a fuzzy-compact primary information.

Table 2

№	Membership function	Proof of fuzzy compactness of primary information
1.	$\mu(x) = e^{-k\lVert x\rVert}$	$\forall\alpha\in(0,1],\ k>1,\ 0\le x<\infty \quad A_\alpha(x) = \{x : \mu(x)\ge\alpha\} \Rightarrow$ $A_\alpha(x) = \{x : e^{-k\lVert x\rVert} \ge \alpha\} = \{x : -k\lVert x\rVert \ge \ln\alpha\} =$ $\left\{x : \lVert x\rVert \le -\frac{\ln\alpha}{k}\right\} = \ \{x : \lVert x\rVert < \varepsilon(\alpha)\},\ \varepsilon(\alpha) = -\frac{\ln\alpha}{k}$
2.	$\mu(x) = e^{-k\lVert x\rVert^2}$	$\forall\alpha\in(0,1],\ k>0,\ A_\alpha(x) = \{x : \mu(x)\ge\alpha\} \Rightarrow$ $A_\alpha(x) = \{x : e^{-k\lVert x\rVert^2} \ge \alpha\} = \{x : -k\lVert x\rVert^2 \ge \ln\alpha\} =$

		$\left\{x:\|x\|^2 \geq -\frac{\ln\alpha}{k}\right\} = \left\{x:\|x\| \leq \sqrt{-\frac{\ln\alpha}{k}}\right\} =$ $\left\{x:\|x\| < \varepsilon(\alpha)\right\},\ \varepsilon(\alpha) = \sqrt{-\frac{\ln\alpha}{k}}$
3.	$\mu(x) = \frac{1}{1+k\|x\|^2}$	$\forall\alpha\in(0,1],\ k>1,\ \ A_\alpha(x) = \{x:\mu(x)\geq\alpha\} \Rightarrow$ $A_\alpha(x) = \left\{x:\frac{1}{1+k\|x\|^2}\geq\alpha\right\} = \left\{x:1+k\|x\|^2 \leq \frac{1}{\alpha}\right\} =$ $\left\{x:\|x\|^2 \leq \frac{1-\alpha}{k\alpha}\right\} = \left\{x:\|x\| \leq \sqrt{\frac{1-\alpha}{k\alpha}}\right\} =$ $\left\{x:\|x\| < \varepsilon(\alpha)\right\}, \varepsilon(\alpha) = \sqrt{\frac{1-\alpha}{k\alpha}}$

Here

$$\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}\ .$$

The statement is proven.

Finding solutions to equations

$$AB = Y$$

comes to the task of finding a fuzzy solution of this equation.

Definition 3. Fuzzy solution of equation $AB = Y$ is called the primary information represented by a fuzzy set $\bigcup_\alpha \alpha A_\alpha$, possessing the following properties:

* operator A and source data B are given;

* $\forall\alpha\in(0,1],\ \mathrm{A}_\alpha = \{B:\mu_A(B)\geq\alpha\}$;

$$\exists\varepsilon(\alpha)>0,\ \sup_{B\in \mathrm{A}_\alpha} \rho_{\mathrm{B}}\left(A(B), A_\alpha\right) < \varepsilon(\alpha) < \infty\ .$$

Here ρ_B – interval between sets *A(B)* and A_α.

Definition 4. Fuzzy solution will be called stable if $\lim_{\alpha\to\sup_{x\in X}\mu(x)} \varepsilon(\alpha) = 0$ and operator $A: B\to Y$ continuous in $B\in Z$.

Statement 2. A fuzzy solution of equations (4) with the membership function of the input variables listed in Table. 3 is sustainable:

Table 3

№	Membership function	Proof of the stability of the primary information
1.	$\mu(B) = e^{-k\|AB-Y\|}$	$\forall\alpha\in(0,1],\ k>1$, $\varepsilon(\alpha) = -\frac{\ln\alpha}{k}$, $\lim_{\alpha\to1}\varepsilon(\alpha) = \lim_{\alpha\to1}\left(-\frac{\ln\alpha}{k}\right) = 0$
2.	$\mu(B) = e^{-k\|AB-Y\|^2}$	$\forall\alpha\in(0,1],\ k>0$, $\varepsilon(\alpha) = \sqrt{-\frac{\ln\alpha}{k}}$,

		$\lim_{\alpha\to1}\varepsilon(\alpha)=\lim_{\alpha\to1}\left(\sqrt{-\frac{\ln\alpha}{k}}\right)=0$
3.	$\mu(B)=\frac{1}{1+k\lVert AB-Y\rVert^2}$	$\forall\alpha\in(0,1],\ k>1$, $\varepsilon(\alpha)=\sqrt{\frac{1-\alpha}{k\alpha}}$, $\lim_{\alpha\to1}\varepsilon(\alpha)=\lim_{\alpha\to1}\left(\sqrt{\frac{1-\alpha}{ka}}\right)=0$
4.	$\mu(B)=\begin{cases}0, & -\infty<(AB-Y)\le-\frac{1}{\sqrt[k]{a}},\\ 1-a\left(-(AB-Y)^k\right), & -\frac{1}{\sqrt[k]{a}}\le(AB-Y)\le0,\\ 1-a(AB-Y)^k, & 0\le(AB-Y)\le\frac{1}{\sqrt[k]{a}},\\ 0, & \frac{1}{\sqrt[k]{a}}\le(AB-Y)<\infty.\end{cases}$	$\forall\alpha\in(0,1],\ -\frac{1}{\sqrt[k]{a}}\le(AB-Y)\le\frac{1}{\sqrt[k]{a}}$ $\varepsilon(\alpha)=\sqrt[k]{\frac{1-\alpha}{a}}$, $\lim_{\alpha\to1}\varepsilon(\alpha)=\lim_{\alpha\to1}\left(\sqrt[k]{\frac{1-\alpha}{a}}\right)=0$
5.	$\mu(z)=\begin{cases}0, & -\infty<(AB-Y)\le-a_2,\\ \frac{a_2+(AB-Y)}{a_2-a_1}, & -a_2\le(AB-Y)\le-a_1,\\ 1, & -a_1\le(AB-Y)\le a_1\\ \frac{a_2-(AB-Y)}{a_2-a_1}, & a_1\le(AB-Y)\le a_2,\\ 0, & a_2\le(AB-Y)<\infty.\end{cases}$	$\forall\alpha\in(0,1],\ -a_2\le(AB-Y)\le a_2$, $\varepsilon(\alpha)=a_2-(a_2-a_1)\alpha$, Equals to $\lim_{\alpha\to1}\varepsilon(\alpha)=\lim_{\alpha\to1}\left(a_2-(a_2-a_1)\alpha\right)=0$ only if $a_1\to0$

Statement 3. Let the operator $A:B\to Y$ continuous in $B\in Z$, then it is possible to construct a stable fuzzy solution with the membership function given in Table.4.

Table 4

№	Membership function	Proof of the possibility of constructing a fuzzy-sustainable solution of primary information
1.	$\mu(B)=e^{-k\lVert AB-Y\rVert}$	$\forall\alpha\in(0,1],\ k>1,\ 0\le B<\infty$, $\varepsilon(\alpha)=-\frac{\ln\alpha}{k}$. It can be built a fuzzy-stable solution in the form $A_\alpha=O_{\varepsilon(\alpha)}(AB)$, $\bigcup_\alpha\alpha A_\alpha$, $\lim_{\alpha\to1}\varepsilon(\alpha)=0$
2.	$\mu(B)=e^{-k\lVert AB-Y\rVert^2}$	$\forall\alpha\in(0,1],\ k>1$, $\varepsilon(\alpha)=\sqrt{-\frac{\ln\alpha}{k}}$. It can be built a fuzzy-stable solution in the form $A_\alpha=O_{\varepsilon(\alpha)}(AB)$, $\bigcup_\alpha\alpha A_\alpha$, $\lim_{\alpha\to1}\varepsilon(\alpha)=0$
3.	$\mu(B)=\frac{1}{1+k\lVert AB-Y\rVert^2}$	$\forall\alpha\in(0,1],\ k>1$, $\varepsilon(\alpha)=\sqrt{\frac{1-\alpha}{k\alpha}}$. It can be built a fuzzy-stable solution in the form $A_\alpha=O_{\varepsilon(\alpha)}(AB)$, $\bigcup_\alpha\alpha A_\alpha$, $\lim_{\alpha\to1}\varepsilon(\alpha)=0$

4.	$\mu(z)=\begin{cases}0, & -\infty<(AB-Y)\le-\frac{1}{\sqrt[k]{a}},\\ 1-a\left(-(AB-Y)^k\right), & -\frac{1}{\sqrt[k]{a}}\le(AB-Y)\le0,\\ 1-a(AB-Y)^k, & 0\le(AB-Y)\le\frac{1}{\sqrt[k]{a}},\\ 0, & \frac{1}{\sqrt[k]{a}}\le(AB-Y)<\infty.\end{cases}$	$\forall\alpha\in(0,1]$, $-\frac{1}{\sqrt[k]{a}}\le B\le0$, $0\le B\le\frac{1}{\sqrt[k]{a}}$, $\varepsilon(\alpha)=\sqrt[k]{\frac{1-\alpha}{a}}$. It can be built a fuzzy-stable solution in the form $A_\alpha=O_{\varepsilon(\alpha)}(AB)$, $\bigcup_\alpha \alpha A_\alpha$, $\lim_{\alpha\to1}\varepsilon(\alpha)=0$
5.	$\mu(z)=\begin{cases}0, & -\infty<(AB-Y)\le-a_2,\\ \frac{a_2+(AB-Y)}{a_2-a_1}, & -a_2\le(AB-Y)\le-a_1,\\ 1, & -a_1\le(AB-Y)\le a_1\\ \frac{a_2-(AB-Y)}{a_2-a_1}, & a_1\le(AB-Y)\le a_2,\\ 0, & a_2\le(AB-Y)<\infty.\end{cases}$	$\forall\alpha\in(0,1]$, $-a_2\le B<-a_1$, $a_1\le B<a_2$ $\varepsilon(\alpha)=a_2-(a_2-a_1)\alpha$. It can be built a fuzzy-stable solution in the form $A_\alpha=O_{\varepsilon(\alpha)}(AB)$, $\bigcup_\alpha \alpha A_\alpha$, $\lim_{\alpha\to1}\varepsilon(\alpha)=0$

Above mechanism for constructing fuzzy-correct models can be used to solve problems of parametric identification, classification, clustering and prediction. [1].

6. Training and adaptation of parameters of fuzzy task models

Process of setting the parameters of the membership functions in the form of Gauss, parabola, triangle, trapezium and bell-shaped form based on neural networks and bee swarm algorithms is considered.

Configuring the parameters of the intellectual analysis model of the state of poorly formalized processes based on neural networks:

The system of recurrent relations for various types of membership functions is used to minimize the criterion

$$E=\frac{1}{2}\sum_{j=1}^{M}\left(y_j-\hat{y}_j\right)^2\to\min, \tag{9}$$

used for training in the theory of neural networks (table 5).

Table 5

Recurrence relations settings of the model parameters of various types of membership functions

Membership function	Recurrent relationships
Gauss form: $\mu(x)=\exp\left(-\left(\frac{x-c}{\sigma}\right)^2\right)$	$c_i^{jp}(t+1)=c_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\frac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\frac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\frac{2(x_i^*-c_i^{jp})\cdot\mu_i^{jp}(x_i^*)}{(\sigma_i^{jp})^2}$, $\sigma_i^{jp}(t+1)=\sigma_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\frac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\frac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\frac{2(x_i^*-c_i^{jp})^2\cdot\mu_i^{jp}(x_i^*)}{(\sigma_i^{jp})^3}$.

Bell shaped form: $\mu(x)=\dfrac{1}{1+\left(\dfrac{x-c}{\sigma}\right)^2}$	$c_i^{jp}(t+1)=c_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{2\left(\sigma_i^{jp}\right)^2(x_i^*-c_i^{jp})}{\left((\sigma_i^{jp})^2+(x_i^*-c_i^{jp})^2\right)^2},$ $\sigma_i^{jp}(t+1)=\sigma_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{2\sigma_i^{jp}(x_i^*-c_i^{jp})^2}{\left((\sigma_i^{jp})^2+(x_i^*-c_i^{jp})^2\right)^2}.$
In the shape of a parabola: $\mu(x)=1-\left(\dfrac{x-c}{\sigma}\right)^2$	$c_i^{jp}(t+1)=c_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{2(x_i^*-c_i^{jp})}{(\sigma_i^{jp})^2},$ $\sigma_i^{jp}(t+1)=\sigma_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{2(x_i^*-c_i^{jp})^2}{(\sigma_i^{jp})^3}.$
In the shape of a triangle: $\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\le x\le b,\\ \dfrac{x-c}{b-c}, & b\le x\le c,\\ 0, \textit{ in other cases.}\end{cases}$	$a_i^{jp}(t+1)=a_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{x_i-b_i^{jp}}{(b_i^{jp}-a_i^{jp})^2},$ $c_i^{jp}(t+1)=c_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{x_i-b_i^{jp}}{(b_i^{jp}-c_i^{jp})^2},$ if $a\le x\le b$, then $b_i^{jp}(t+1)=b_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d}_j\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{a_i^{jp}-x_i}{(b_i^{jp}-a_i^{jp})^2},$ If $b\le x\le c$, then

	$b_i^{jp}(t+1)=b_i^{jp}(t)-\eta\left(y_t-\hat{y}_t\right)w_{jp}\dfrac{\prod_{i=1}^{n}\mu^{jp}(x_i)}{\mu^{jp}(x_i)}\times$ $\times\dfrac{\overline{d_j}\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\dfrac{c_i^{jp}-x_i}{(b_i^{jp}-c_i^{jp})^2}.$

$$w_{jp}(t+1)=w_{jp}(t)-$$
$$-\mu\left(y_t-\hat{y}_t\right)\frac{\overline{d_j}\sum_{j=1}^{m}\mu^{d_j}(y)-\sum_{j=1}^{m}\overline{d}_j\mu^{d_j}(y)}{\left(\sum_{j=1}^{m}\mu^{d_j}(y)\right)^2}\cdot$$
$$\cdot w_{jp}\prod_{i=1}^{n}\mu^{jp}(x_i).$$

2. The process of setting the parameters of a fuzzy model of intellectual analysis of the state of poorly formalized processes based on an evolutionary algorithm — a bee swarm is considered. This algorithm was developed by analogy with the behavior of wasps in a colony of bees.

The main essence of the model parameter settings based on the bee swarm algorithm is the choice of parameter values that minimize the difference between the real properties of the object and the output results of the model. This algorithm consists of the following steps.

Step 1. Initialization. Here totalNumberBees is the number of bees, numberInactive is the number of inactive bees, numberScout is the number of scout bees, maxNumberVisits is the number of visits to nectar sources, maxNumberCycles is the number of iterations, the intervals of parameter values are determined a, b, c and w.

Step 2. Scouts survey the area around the hive in search of new sources of nectar. In this case, the initial values of the parameters are determined, and the results are stored in the BS matrix.

Step 3. Waggle dance – dance of watching bees. Here, from the sources of nectar found, the most optimal (in which there are many nectars or the closest ones) are transferred from the BS matrix to the WG matrix.

Duration Waggle dance is determined by the formula $D_i = d_i A$. Here A is the scalability factor; value indicating the relative utility, quality and volume of nectar found d_i - dancing *i* – intelligence bee.

After selecting the necessary source of nectar, the worker bee begins its flight toward nectar.

Step 4. On the basis of the WG matrix obtained from scouting bees, worker bees carry nectar and find new sources (parameter values) around the source of this nectar. The information found is entered into the NW matrix.

Step 5. Based on the WG information, reconnaissance bees transfer nectar and determine the result that gives the most optimal values, which is assigned to the variable best. The results obtained are entered into the NB matrix.

Step 6. Forming an archive of solutions based on existing matrices NW, NB, WG.

Step 7. Under the conditions of the criterion $E=\frac{1}{2}\sum_{j=1}^{M}(f_j(w,a,b,c,d)-\hat{y}_j)^2\to\min$ or performing a certain iteration up to maxNumberCycles determines the optimal values of the parameters from WG.

Here $f_j\left(w,a,b,c,d\right)$ - model output, *w* – weight rules, $\hat{y}_j$ - real characteristics of the object, a, b, and c are the parameters of the membership functions. These parameters are determined according to the type of the membership function. If the membership function is in the form of Gauss, parabola, bell-shaped form, then the parameters of the functions will be a and b. If the membership function is in the form of a trapezoid, then the parameters of the functions will be in the form of a, b, c and d.

Step 8. If the conditions of the corresponding criterion are not fulfilled, go to step 2.

With each iteration of the algorithm, the values of the model parameters approach the optimal ones.

Conclusion

Analysis of the tasks of fuzzy multicriteria optimization, formed when building a fuzzy model of intellectual analysis of the state of poorly formalized processes, allows solving problems of multicriteria optimization that arise when building models of classification, evaluation and forecasting of the state of processes in conditions of fuzzy information.

Experimental studies have shown a higher efficiency of the developed algorithms in comparison with the known algorithms in solving model problems of classification, evaluation and forecasting.

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