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# MULTI-CRITERIAL OPTIMIZATION OF INFORMATION GRANULES IN FUZZY IF-THEN RULES

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***Abstract:*** *Investigation of trade-off between characteristics of information granules for construction of fuzzy rule-based model is challenging scientific problem. Main optimization requirement in information granules estimation (antecedent and consequent parts of fuzzy rules) are cardinality which characterizes justifiability (reliability) of information granules in light of the available experimental data and specificity that characterizes semantics of information granules. This paper is devoted to a multi-criterial approach to solving of optimization problem of construction of interval and fuzzy information granules. Validity of the proposed approach is proved by numerical examples.*

***Keywords:*** *information granularity, cardinality, specificity, interval, triangular fuzzy number.*

## 1. Introduction

The problem of processing information granules appears in a wide range of applications. The main optimality criteria describing quality of information granularity include cardinality and specificity measures[1,8].

Cardinality reflects coverage of evaluation-relevant values and pays to abstraction of information granule.

Specificity concerns level of concentration of evaluation-relevant values expressed by information granule. In other words, the specificity can be considered as a level of a detail provided.

It is obvious that specificity of information granule decreases with the increase of level of abstraction. We may loss a reliability of evaluation with increase of a level of details of an evaluation and vice versa. Thus, determination of an optimal information granulation becomes an interesting and context dependent practical problem.

Let us shortly overview existing works in this area.

In [5] the author introduces two measures that are of practical relevance and offer a useful insight into the nature and further usage of the granule in various constructs.

In general, specificity measure satisfies boundary conditions sp({x})=1 (information granule is most specific if it is a single element), sp(X)=0 (V is least specific if it is a universe of discourse) and monotonicity, that is $sp(A) \geq sp(B)$ when $A \subset B$. This reflects our intuition that the more detailed information granule comes with the higher specificity.

In [6] granular representatives of experimental data as fuzzy sets are given. Maximization of the coverage of experimental data and minimization of the spread of the fuzzy set are discussed.

The principle of minimal specificity is considered in [2]. In this paper possibilistic approach to investigation of specificity is considered. It is shown that principle of minimal specificity leads to certainty rule.

Trade-off between specificity and granularity is investigated in [9].

The specificity of fuzzy sets measures is the degree to which the set designates a unique element of the set. The set of fuzzy rules are measured by the specificity of the fuzzy sets in the consequents of the rules. A lower specificity indicates that the consequent provides a wider range of positive values. In particular when the output domain is partitioned by triangular fuzzy sets with peak points $\{c_1, c_2, ..., c_n\}$, the specificity of the rule “IF X is $A_i$ then Z is $C_k$” is the distance $[C_{k-1} - C_{k+1}]$ between neighboring peak points of the output domain decomposition.

In [4] they consider construction of fuzzy granules in rule-based control system to achieve predefined specificity of huge fuzzy rule base.

In [3] a systematic study of specificity of fuzzy IF … THEN rules is proposed. A new measure of specificity is introduced.

In this paper multi-criterial approach to optimal construction of interval and fuzzy granules in fuzzy rule-base is considered.

The rest of this paper is organized as follows. Section 2 includes necessary formal concepts used in this paper. In Section 3 state-of the multi-criterial optimization problem for interval and fuzzy granules construction is formulated. In Section 4 the solution method for the problem formulated in Section 3 is proposed. Section 5 includes numerical examples. Section 6 concludes.

## 2. Preliminaries

**Definition 1. Fuzzy number**

A fuzzy number is a fuzzy set A on R which possesses the following properties:

a) A is a normal fuzzy set;

b) A is a convex fuzzy set;

c) α-cut of A, $A^{\alpha}$ is a closed interval for every $\alpha \in (0,1]$;

d) the support of A, $A^{+0}$ is bounded.

**Definition 2. Specificity of fuzzy number**

The specificity measure defined on the basis of cardinality concept is as follows[7]:

$$Sp(A) = \int_0^{hgt(A)} \frac{1}{|A^{\mu}|} d\mu \tag{1}$$

where $|A^{\mu}|$ is a cardinality of $\mu$ -cut of $A$.

Denote $[x_{\min}, x_{\max}] \subset R$ a universal set. Specificity and cardinality of interval $[a,b] \subset [x_{\min}, x_{\max}]$ are defined below.

**Definition 3. Cardinality of interval**

The cardinality of interval $C([a,b])$, $[a,b] \subset [x_{\min}, x_{\max}]$ is defined as an are constrained by the characteristic functon $I_{[a,b]}$ of interval:

$$C([a,b]) = \int_{x_{\min}}^{x_{\max}} I_{[a,b]}(x)dx = (b-a)$$

**Definition 4. Specificity of interval**

The specificity of interval is defined $Sp([a,b])$, $[a,b] \subset [x_{\min}, x_{\max}]$ as follows

$$Sp([a,b]) = \frac{b-a}{x_{\max} - x_{\min}}$$

**Definition 5. Triangular Membership Functions**

In real-world problems, the form of the membership functions (MFs) usually is chosen depending on how it is reflective to the problem at hand. Triangular MFs are the simplest model of the fuzzy sets and are characterized only by three parameters. Analytical representation of triangular MF is given as follows. Also see graphical representation in Figure 1.

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{if } \ a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{if } \ a_2 \le x \le a_3 \\ 0 & \textit{otherwise} \end{cases}$$

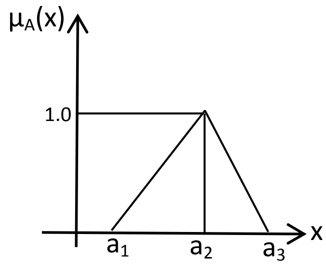


Figure 1. **A triangular MF**

## 3. Statement of problem

Let us consider a problem of construction of a granule that satisfies a desired tradeoff between specificity and cardinality criteria. Indeed, these important criteria are conflicting. Specificity pays to decreasing of imprecision of information. Cardinality pays to increasing of reliability of information.

We will consider two types of granule: interval granule and fuzzy granule.

Graphical representation of an interval granule is shown in Figure 2.

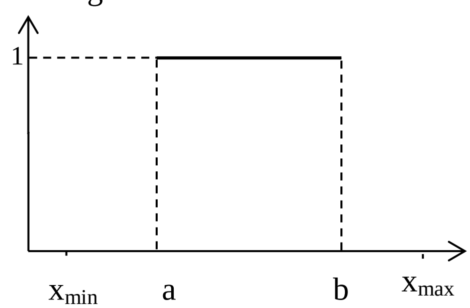


*Figure 2.* ***An interval granule.***

The problem of construction of an interval granule $[a,b] \subset R$ is formulated as follows.

$$\max(C([a,b]), Sp([a,b])) \qquad (2)$$

s.t.

$$\begin{aligned} &x_{\min} < a < x_{\max}, \\ &x_{\min} < b < x_{\max}, \\ &a < b, \\ &b - a > l, l > 0 \end{aligned} \qquad (3)$$

The last constraint restricts the length of an interval granule from below to rule out its reduction to a point.

The problem of a fuzzy granule construction is formulated as follows.

$$\max(C(A) = \int_{x_{\min}}^{x_{\max}} A(x)dx), \qquad (4)$$

$$max(Sp(A) = \int_0^1 \left(1 - \frac{h(\alpha)}{x_{\max} - x_{\min}}\right) d\alpha)) \qquad (5)$$

s.t.

$$A : [x_{\min}, x_{\max}] \to [0,1]$$

$$\underset{\alpha>0}{h(\alpha)} = \left(\max\left\{x \middle| A^{-1}(y) = \alpha\right\}\right) - \left(\min\left\{x \middle| A^{-1}(y) = \alpha\right\}\right) \qquad (6)$$

$$\sup pA > l, l > 0$$

The last constraint rules out reduction of a fuzzy granule interval to a point.

Graphical representation of a fuzzy granule is shown in Figure 3.

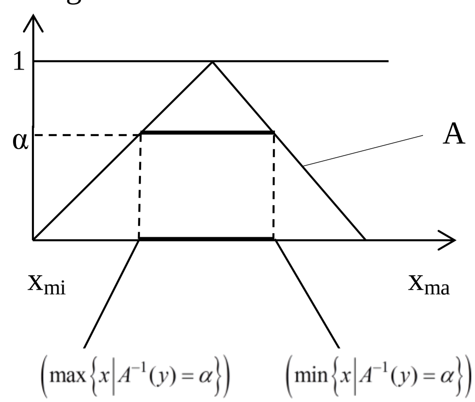


*Figure 3.* ***A fuzzy granule***

## 4. Solution method

As a solution approach for problems (2)-(3) and (4)-(6), we propose using goal programming approach. For problem (4)-(6) it is described as follows.

$$(C(A), Sp(A)) \to (C_g, Sp_g) \qquad (7)$$

s.t.

$$A : [x_{\min}, x_{\max}] \to [0,1] \qquad (8)$$

$C_g, Sp_g$ are desired values of cardinality and specificity of an interval granule assigned by a user to attain a trade-off between the conflicting criteria.

A solution approach for (2)-(3) is formulated analogously and may be considered as a special case of (7)-(8).

## 5. Examples

### 5.1 An interval granule construction

Consider problem (2)-(3) with $[x_{\min}, x_{\max}] = [5,10]$ and goal values $C_g = 1.2, Sp_g = 0,95$. By applying goal programming approach (7)-(8), we have found the solution:

$$C = 1.1, Sp = 0.88,$$
$$a = 5, b = 6.1.$$

As one can see, the obtained interval granule $[a,b] = [5, 6.1]$ is sufficiently acceptable taking into account the closeness of its specificity and cardinality values to the predefined goal values.

The graphical representation of the solution in the space of cardinality and specificity criteria is shown in Figure 4.

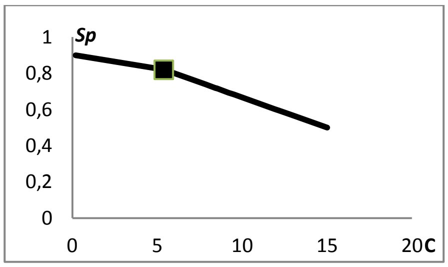


*Figure 4.* ***The front of the cardinality and specificity criteria for interval granules***

## 5.2.A fuzzy granule construction

Consider problem of a fuzzy granule construction (4)-(6) with $[x_{\min}, x_{\max}] = [0, 30]$ and goal values $C_g = 5, Sp_g = 0,9$. For purpose of computational simplicity, we will consider construction of a fuzzy granule as a triangular fuzzy number $A = (a_1, a_2, a_3)$. By applying goal programming approach (7)-(8), we have found the solution:

$$C = 5.4, Sp = 0.82,$$
$$A = (18, 23, 28.8).$$

The solution is shown in Figure 5.

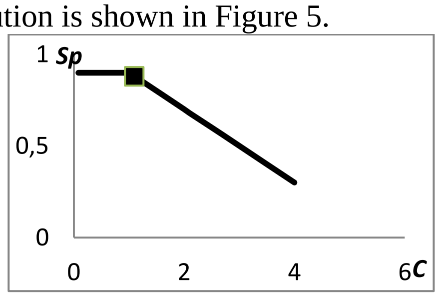


*Figure 5.* ***Dependence between cardinality and specificity criteria (for fuzzy granules) and the goal values***

It can be seen that the specificity and cardinality of the obtained fuzzy granule $A = (18, 23, 28.8)$ are close to the predefined goal values.

## 6. Conclusion

Main optimization requirement in information granules estimation (antecedent and consequent parts of fuzzy rules) relies on cardinality which characteristics justifiability of information granules and specificity which measures a level of details.

Solution of this optimization problem with conflicting criteria is challenging scientific problem. In this study we have formulated this problem and proposed a solution approach that relies on multiobjective goal programming. Numerical examples and computer simulations are used to illustrate acceptability and applicability of the suggested approach to determine an optimal information granularity.

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