На сегодняшний день успешно определены правовые и организационно-технические механизмы создания системы «Безопасный город» Республики Узбекистан, позволяющей оперативно и эффективно реагировать на современные вызовы и угрозы общественной безопасности.

В успешном выполнении поставленной задачи необходимо активное участие не только государственных органов, но и институтов гражданского общества, научно-исследовательских и научно-производственных объединений и граждан Республики Узбекистан. [9]

Литература

1.Российская академия наук Сибирское отделение институт вычислительной математики и математической геофизики Аксель Иванович Берг ответственный редактор академик А.С. Алексеев Москва «Наука» 2007 с.6

2.Полежаев А.П., Минаев В.А., Абдусагатов К.Х. Построение модели распределения ресурсов в системах организационного управления. –М.: Академия МВД СССР, 1988.

3.Постановление Президента Республики Узбекистан от 29 августа 2017 года № 3245 «О мерах по дальнейшему совершенствованию систем управления проектами в сфере информационно-коммуникационных технологий»

4. Концепции создания и дальнейшего развития системы «Безопасный город»

5. www.keepertele.com

6. Keith D. Harries. Mapping crime: principle and practice – U.S.Dept. of Justice Programs, National Institute of Justice, 1999 г. С.1-2.

7. Географическая информационная система [Электронный ресурс]. Электрон.дан. Режим доступа: http://www.wikipedia.org/wiki (дата обращения: 15.04.2014).

8. Журкин И.Г., Шайтура С.В. Геоинформационные системы. – М.,2009г. С.6-17.

9.Программа поэтапных мер по созданию системы «Безопасный город».

Information systems in management of the complex "Safe city"

Abstract: The article considers some general issues of constructing a hybrid automated system - "Safe City" - for solving the main emergency and technical tasks of the city economy in the light of the requirements of the Resolution of the President of the Republic of Uzbekistan dated August 29, 2017 No. 3245" About the measures for the further improvement of project management systems in the field of information and communication technologies"

Keywords: management, information systems, information-analytical function, geoinformation system, safe city

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ON THE LINEAR EVASION DIFFERENTIAL GAMES OF MANY PURSUERS AND ONE EVADER WITH INTEGRAL CONSTRAINTS

We study a linear evasion differential game of one evader from many pursuers with integral constraints on control functions of players. The terminal set is union of a finite number of subspaces. The critical case where "rotatability" condition fails to hold is studied. More precisely, when projections of control sets are segments parallel to coordinate axes, effectively verifiable sufficient conditions of evasion are obtained. The evasion is sequentially implemented from each of pursuers. To construct the evasion control, the initial positions, current states and controls of pursuers are used.

Keywords: Linear differential game, evasion, many pursuers, integral constraint, critical case.

1. Introduction

There are many works devoted to evasion differential games (see, for example, [1–21]). The problem of evasion from any initial position on the semi-infinite time interval first was formulated and solved for a linear differential game in [8-9]. Results of these works were further extended to the case of integral constraints [1] and [5]. In the works [1–2], [6], [5-7], [10-13], [19,20,21] evasion differential game problems of various structure were studied and solved under assumption that the conditions of" rotatability" and "advantage" are satisfied in one or another form.

In [20], sufficient conditions of evasion were obtained when the projection of control set of evader is a segment not parallel to coordinate axes. In [8–9] first were investigated the critical case, when the projection of control set of evader is a segment parallel to one of the coordinate axes. In those papers, evasion differential games of one evader and one pursuer were investigated and new sufficient conditions of evasion were obtained under information discrimination of the pursuer. The works [15-18] were devoted to investigation and improvement of sufficient conditions of evasion in critical case [8–9]. In [15-16], a method of evasion in direction was proposed and sufficient conditions of evasion in critical case were obtained, and in [17] sufficient conditions of evasion from many pursuers with integral constraints on controls were obtained in critical case.

A method of alternating directions was proposed in [2] for nonlinear evasion differential games, which enables to reduce dimensionality of the subspace where the evader needs a directional advantage. In [4] a linear differential game with geometric constraints was studied when the Pontryagin condition is not satisfied. A sufficient condition of evasion was proposed.

In the present paper, effective sufficient conditions of evasion were obtained, which improves the results of the work [17] and, moreover, includes a new class of differential games. The present paper is closely allied to studies [15-18].

2. Statement of Problem

Consider a differential game in R^{n_i} described by the following equations

$$\dot z_i = C_i z_i - B_i u_i + D_i v + a_i \qquad (1)$$

where C_i, B_i, D_i -are constant matrices with appropriate dimensions, $u_i \in R^{p_{1i}}$, $v \in R^{q_1}$ are control parameters of the i-th pursuer and evader, respectively, subjected to constraints

$$\|u_i(\cdot)\|_{L_2} \le \rho_i, \qquad (2)$$

$$\|v(\cdot)\|_{L_2} \le \sigma. \qquad (3)$$

Differential game is started beginning at the time $t = 0$ from the initial point $z_0 = (z_{10}, z_{20}, \cdots, z_{m0})$, $z_{i0} \notin M_i$, and differential game is said to be completed if $z_i(\tau) \in M_i$ at some $i \in \{1,...,m\}$ and $\tau > 0$, where M_i is terminal set, which is a linear subspace of R^{n_i}.

Definition 1. Measurable functions $u_i = u_i(\cdot)$ and $v = v(\cdot)$ that satisfy conditions (2) , (3) , are called admissible controls of the i-th pursuer and evader respectively.

Definition 2. A function $v = v(t, z_0, z_1, u_1, u_2, ..., u_m)$ is called strategy of evader if for any controls of the pursuers $u_i = u_i(t)$, $i = 1,2,\dots,m,$ the initial value problem

$$\dot z_i = C_i z_i - B_i u_i(t) + D_i v(t) + a_i$$
$$z_i(0) = z_{i0}, \; i = 1,2,\dots,m. \qquad (4)$$

has a unique solution $z_1(t),\ z_2(t),\ ...,\ z_m(t),\ t \ge 0,$ and the following inequality

$$\int_0^\infty |v(t, z_0, z, u_1, u_2, \dots, u_m)|^2 dt \le \sigma^2$$, holds true.

By a solution of the initial value problem (4), we mean m absolutely continuous functions $z_1(t),\ z_2(t),\ \dots,\ z_m(t),\ t \ge 0$, that satisfy initial conditions in (4) and differential equations in (4) almost everywhere on $[0, \infty)$.

Definition 3. We say that evasion is possible in the game (1)–(3) from initial position $z_0 = (z_{10}, z_{20}, \cdots, z_{m0})$, with $z_{i0} \notin M_i$, $i = 1,2,\dots,m$, if for any controls of the pursuers, there is a strategy of evader such that, for all $t > 0$ and $i = 1,2,\dots,m$, $z_i(t) \notin M_i$.

Definition 4. We say that evasion is possible in the game in the game (1) -(3), if evasion is possible in the game (1) -(3) from all initial positions

$$z_0 = (z_{10}, z_{20}, \cdots, z_{m0}),\ z_{i0} \notin M_i.$$

Problem. Find a condition of evasion in the game (1) -(3).

3. Main result

Let L_i be the orthogonal complement of M_i in R^{n_i} and W_{i1}, W_{i2} be one-dimensional subspaces in L_i. Denote by π_{i1}, π_{i2} the orthogonal projection operators of R^{n_i} to W_{i1}, W_{i2} respectively. Let $F_{ij} : W_{ij} \to R^1$, $j = 1,2$, $i \in \{1,2,\dots,m\}$, be linear one to one maps.

To formulate the main results, we make some assumptions.

Assumption 1. There exist positive integers k_i, one-dimensional subspaces W_{i1}, W_{i2}, and linear one to one maps $F_{ij} : W_{ij} \to R^1$, $j = 1,2$, such that

(a) for any $i \in \{1,2,\dots,m\}$ and $s_j = 0,1,\dots,k_{ij} - 2$, $k_{i1} = k_i$, $k_{i2} = n_i - r_i - 1$, the set $F_{ij} \pi_{ij} C_i^{s_j} (B_i R^{p_i} + D_i R^q)$ is singleton, and $C_i^{r_i} \ne 0$, where r_i is multiplicity of zero root of characteristic equation of the matrix C_i.

(b) $F_{i1} \pi_{i1} C_i^{k_i - 1} D_i R^q = R^1$, $\forall\ i \in \{1,2,\dots,m\}$

Comments to conditions Assumption1:

-in Assumption1(a) the condition $k_{i1} = k_i$ shows the degree «inertness» of players and means that degree of «inertness» of the evader is equal to the can't be more than degree of inertness of each of pursuers;

-the condition $k_{i2} = n_i - r_i + 1$ means the presence of condition «critical case» [12,13,22-24,26,28];

-the condition (b) means that the evader has an advantage running away over pursuers.

By the Hamilton – Kayley theorem, any matrix satisfies its characteristic equation. In our case, from condition (a) we have

$$C_i^{r_i}(b_0 C_i^{n_i - r_i} + b_1 C_i^{n_i - r_i - 1} + \cdots + b_r) = 0, \qquad (5)$$

Since $C_i^{r_i} \neq 0$, applying the maps $F_{i2}\pi_{i2}$ to (5) we find that

$$b_0 F_{i2}\pi_{i2} C_i^{n_i - r_i} = b_1 F_{i2}\pi_{i2} C_i^{n_i - r_i - 1} + \cdots + b_r, \qquad (6)$$

Therefore, Assumption 1(a) and (6) implies that the set $F_{ij}\pi_{ij}C_i^{\ell}(B_i R^{p_i} + D_i R^q)$ is a singleton for all $\ell \geq n_i - r_i$. Clearly, $F_{ij}\pi_{ij}C_i^{\ell}(B_i R^{p_i} + D_i R^q) = \{0\}$

Let

$$a_{ij} = F_{i2}\pi_{i2}C_i^{\ell} a_i, \ell = 0,1,\ldots,$$

$$\Phi_i(z_i) = \sum_{j=0}^{n_i} (F_{i2}\pi_{i2}C_i^j z_i - a_{ij-1})^2, a_{i,-1} = 0.$$

Define the matrices H_i, H_i^* as follows:

$$H_i = \begin{bmatrix} F_{i2}\pi_{i2}E_i \\ F_{i2}\pi_{i2}C_i \\ F_{i2}\pi_{i2}C_i^2 \\ \cdots\cdots\cdots \\ F_{i2}\pi_{i2}C_i^{n_i} \end{bmatrix}, \ H_i^* = \begin{bmatrix} F_{i2}\pi_{i2}E_i & 0 \\ F_{i2}\pi_{i2}C_i & a_{i0} \\ F_{i2}\pi_{i2}C_i^2 & a_{i1} \\ \cdots\cdots\cdots & \\ F_{i2}\pi_{i2}C_i^{n_i} & a_{in_i-1} \end{bmatrix}, \qquad (7)$$

where E_i is $n_i \times n_i$ identity matrix.

Assumption 2. $rankH_i < rankH_i^*$ for all $i \in \{1,2,\ldots,m\}$.

According to Assumption 1 (b) $F_{i1}\pi_{i1}C_i^{k_i-1}B_i R^{p_i} \subset F_{i1}\pi_{i1}C_i^{k_i-1}D_i R^q$ therefore, there are linear maps $G_i : R^{p_i} \to R^q$ such that $F_{i1}\pi_{i1}C_i^{k_i-1}B_i = F_{i1}\pi_{i1}C_i^{k_i-1}D_i G_i$.

Let

$$\alpha_i = \inf\{\|G_i\| / F_{i1}\pi_{i1}C_i^{k_i-1}B_i = F_{i1}\pi_{i1}C_i^{k_i-1}D_i G_i\}.$$

Assumption 3.

$$\sigma^2 > \alpha_1^2\rho_1^2 + \alpha_2^2\rho_2^2 + \cdots + \alpha_m^2\rho_m^2.$$

Theorem 1. If Assumptions 1–3 hold, then evasion is possible in the game (1)–(3).

Proof. Define the functions

$$g_i(t, z_i) = F_{i2}\pi_{i2}e^{tC_i}z_i - \sum_{r=0}^{\infty} a_{ir}\frac{t^{r+1}}{(r+1)!},$$

$i \in \{1,2,\ldots,m\}$.

By construction, if z_i is a fixed point in R^{n_i}, then the function $g_i(t, z_i)$ is an analytical function in t on any finite interval. Consider the following system of algebraic equations with unknown vector z_i:

$$\begin{cases} F_{i2}\pi_{i2}z_i = 0 \\ F_{i2}\pi_{i2}C_i z_i = a_{i0} \\ F_{i2}\pi_{i2}C_i^2 z_i = a_{i1} \\ \cdots\cdots\cdots \\ F_{i2}\pi_{i2}C_i^{n_i} z_i = a_{in_i-1} \end{cases} \qquad (8)$$

Assumption 2 implies that the system (8) has no solution with respect to . Then, comparing coefficients of the function $g_i(t, z_i)$ with the system (8), yields that each function $g_1(t, z_1), g_2(t, z_2), \ldots, g_m(t, z_m)$ is not identically equal to zero on $[0, \infty)$ for fixed $z_i \in R^{n_i}$. Let $\Sigma_i = \{g_i(\cdot, z_i) | \ z_i \in R^{n_i}\}$. Then Σ_i is a finite dimensional family and any non-zero function of the family Σ_i, for fixed z_i, has a finite number of positive zeros on finite interval[1]. Therefore, if $z_{i0} \notin M_i$, all the functions $g_1(t, z_{10}), \ldots, g_m(t, z_{m0})$, have finite number of positive zeros on finite time interval. As a consequence, we obtain if $t_1, t_2, \ldots, t_n, \ldots$, is a sequence with $t_r < t_{r+1}$, consisted of positive zeros of the functions $g_1(t, z_{10}), \ldots, g_m(t, z_{m0})$, then $t_n \to \infty$ as $n \to \infty$, and there exist constants $\theta_r > 0$ such that

$$\varepsilon_1 = t_1 - \theta_1 \geq 0, \ t_r + \theta_r < t_{r+1} - \theta_{r+1}, \ r = 1,2\ldots.$$

Denote

$$I_0 = [0, \varepsilon_1], \ I_r = [t_r - \theta_r, t_r + \theta_r], \ r = 1,2,.$$

Let $u_i(t), v(t), \ 0 \leq t \leq 1$, be arbitrary admissible controls of players. Then by Assumption 1(a) for the solution $z_i(t), \ 0 \leq t \leq 1,$ of the initial value problem

$$\dot{z}_i = C_i z_i - B_i u_i(t) + D_i v(t) + a_i, \ z_i(0) = z_{i0},$$

we have

$$F_{i1}\pi_{i1}z_i(t) = \phi_i(t, z_{i0}) + \int_0^t \frac{(t-\tau)^{k_i-1}}{(k_i-1)!} F_{i1}\pi_{i1}C_i^{k_i-1}[D_i v(\tau) - B_i u_i(\tau)]d\tau + h_i(t, 0)$$

$$F_{i2}\pi_{i2}z_i(t) = g_i(t, z_{i0}), \text{ e}$$

$$\phi_i(t, z_{i0}) = F_{i1}\pi_{i1}e^{tc_i}z_{i0} + \int_0^t F_{i1}\pi_{i1}e^{(t-\tau)C_i}a_i d\tau,$$

$$h_i(t,x)=$$
$$\int_x^t \sum_{r=k_i}^{\infty} \frac{(t-\tau)^r}{r!} F_{i1}\pi_{i1}C_i^r[D_i v(\tau)-B_i u_i(\tau)]d\tau.$$

Using the Cauchy – Schwartz inequality and admissibility of controls of players, we obtain

$$\left|h_i(t,x)\right| \le \frac{d_i}{k_i!}(t-x)^{k_i+1/2},\ 0\le x\le t\le 1, \quad (10)$$

where d_i a positive constant.

Now, we construct strategy for the evader. Set $v(t)\equiv 0$ on the interval I_0. Since $g_i(t,z_i)\ne 0$, $t\in I_0$, for all $i\in\{1,2,...,m\}$, then, clearly, $z_i(t)\notin M_i$, $t\in I_0$. In particular, if we take $z_{i0}=z_i(\varepsilon_1)$, then $z_{i0}\notin M_i$. For $z_i(t)$, $t\in I_1$, we can see from (9) that

$$F_{i1}\pi_{i1}z_i(t)=\phi_i(t,z_{i0})+\int_{\varepsilon_1}^t \frac{(t-\tau)^{k_i-1}}{(k_i-1)!}$$
$$F_{i1}\pi_{i1}C_i^{k_i-1}\left[D_i v(\tau)-B_i u_i(\tau)\right]d\tau+h_i(t,\varepsilon_1),$$
$$F_{i2}\pi_{i2}z_i(t)=g_i(t,z_{i0}). \quad (11)$$

By definition of the greatest lower bound, there are linear maps $G_i: R^{p_i}\to R^q$ such that $\sigma^2>\|G_1\|^2\rho_1^2+...+\|G_m\|^2\rho_m^2$ and $F_{i1}\pi_{i1}C_i^{k_i-1}B_i=F_{i1}\pi_{i1}C_i^{k_i-1}D_iG_i$. Partition the interval I_1 into m subintervals by the numbers $\varepsilon_1<\varepsilon_2<\dots<\varepsilon_m<t_1$, and we define $v(t)$ on $\varepsilon_1\le t\le t_1+\theta_1$ step by step. We specify this numbers later. On the interval $\varepsilon_1\le t\le\varepsilon_2$, we set $v(t)\equiv v_1(t)=G_1u_1(t)+w_1$, where $w_1\in R^q$ is a solution of the equation

$$F_{11}\pi_{11}C_1^{k_1-1}D_1w_1=-\omega_1, \quad (12)$$

where $\omega_1\in\Gamma_1$, $\Gamma_1=\{\omega:\ |\omega|\le b_1,\ b_1>0\}$, while on the segment $\varepsilon_2\le t\le t_1+\theta_1$, $v(t)$ is still considered arbitrary. It follows from Assumption 1(b) that the equation (12) has a solution with respect to the vector $w_1\in R^q$. Let ω_1 be a number from Γ_1, and $w_1\in R^q$ be corresponding lexicographically minimal solution of the equation (12). Then for the solution $z_1(t)$, $\varepsilon_1\le t\le t_1+\theta_1$, with the initial value $z_{10}=z_1(\varepsilon_1)$, we obtain from (11) at $i=1$ the following: for $t\in[\varepsilon_1,\varepsilon_2]$,

$$F_{11}\pi_{11}z_1(t)=\phi_1(t,z_{10})-\frac{(t-\varepsilon_1)^{k_1}}{k_1!}[\omega_1-k_1!\cdot(t-\varepsilon_1)^{-k_1}h_1(t,\varepsilon_1)];$$
$$F_{12}\pi_{12}z_1(t)=g_1(t,z_{10}); \quad (13)$$

and for $t\in[\varepsilon_2,t_1+\theta_1]$,

$$F_{11}\pi_{11}z_1(t)=\phi_1(t,z_{10})-\frac{(t-\varepsilon_1)^{k_1}}{k_1!}[\omega_1-\omega_1\cdot\left(\frac{t-\varepsilon_2}{t-\varepsilon_1}\right)^{k_1}$$
$$-\gamma_1(t)\frac{(t-\varepsilon_2)^{k_1-\frac{1}{2}}}{(t-\varepsilon_1)^{k_1}}-(t-\varepsilon_1)^{-k_1}\cdot k_1!h_1(t,\varepsilon_2)],$$
$$F_{12}\pi_{12}z_1(t)=g_1(t,z_{10}),\ \varepsilon_2\le t\le t_1+\theta_1. \quad (14)$$

where

$$\gamma_1(t)=\int_{\varepsilon_2}^t \frac{(t-\tau)^{k_1-1}}{(k_1-1)!}F_{11}\pi_{11}C_1^{k_1-1}\left[D_1v(\tau)-B_1u_1(\tau)\right]d\tau\cdot(t-\varepsilon_1)^{-k_1+\frac{1}{2}}$$

Next, on the interval $\varepsilon_2\le t\le\varepsilon_3$, we set $v(t)=v_2(t)=G_2u_2(t)+w_2$, where $w_2\in R^q$ is a solution of the equation (15) where $\omega_2\in\Gamma_2$, $\Gamma_2=\{\omega:\ |\omega|\le b_2,\ b_2>0\}$, while on the segment $\varepsilon_3\le t\le t_1+\theta_1$, $v(t)$ is still considered arbitrary. Assumption 1(b) implies that there exists a solution $w_2\in R^q$ of the equation (15). Let ω_2 be a number from Γ_2, and $w_2\in R^q$ be corresponding lexicographically minimal solution of the equation (15). Then for the solution $z_2(t)$, $\varepsilon_2\le t\le t_1+\theta_1$, with the initial condition $z_{20}=z_2(\varepsilon_2)$, letting $i=2$ and using (11) we have the following:

for $t\in[\varepsilon_2,\varepsilon_3]$,

$$F_{21}\pi_{21}z_2(t)=\phi_2(t,z_{20})-\frac{(t-\varepsilon_2)^{k_2}}{k_2!}[\omega_2-k_2!\cdot(t-\varepsilon_2)^{-k_2}h_2(t,\varepsilon_2)],$$
$$F_{22}\pi_{22}z_2(t)=g_2(t,z_{20}); \quad (16)$$

and for $t\in[\varepsilon_3,t_1+\theta_1]$,

$$F_{21}\pi_{21}z_2(t)=\varphi_2(t,z_{20})-\frac{(t-\varepsilon_2)^{k_2}}{k_2!}[\omega_2-\omega_2\left(\frac{t-\varepsilon_3}{t-\varepsilon_2}\right)^{k_2}-\gamma_2(t)\cdot\frac{(t-\varepsilon_3)^{k_2-\frac{1}{2}}}{(t-\varepsilon_2)^{k_2}}-$$
$$-k_2!(t-\varepsilon_2)^{-k_2}h_2(t,\varepsilon_3)];$$
$$F_{22}\pi_{22}z_2(t)=g_2(t,z_{20}),\ \varepsilon_3\le t\le t_1+\theta_1. \quad (17)$$

where

$$\gamma_2(t)=\int_{\varepsilon_3}^{t}\frac{(t-\tau)^{k_2-1}}{(k_2-1)!}F_{21}\pi_{21}C_2^{k_2-1}\left[D_2v(\tau)-B_2u(\tau)\right]d\tau\cdot(t-\varepsilon_2)^{-k_2+\frac{1}{2}}.$$

We continue in this fashion to choose the control $v(t)$ on $\varepsilon_{m-1}\le t\le\varepsilon_m$. Set $v(t)=v_{m-1}(t)=G_{m-1}u_{m-1}(t)+w_{m-1}$, where $w_{m-1}\in R^q$ is a solution of the equation

$$F_{m-1,1}\pi_{m-1,1}C_{m-1}^{k_{m-1}-1}D_{m-1}w_{m-1}=-\omega_{m-1} \qquad (18)$$

where, $\omega_{m-1}\in\Gamma_{m-1}=\left\{\omega:\ |\omega|\le b_{m-1},\ b_{m-1}>0\right\}$, while on the segment $\varepsilon_m\le t\le t_1+\theta_1$, $v(t)$ is still considered arbitrary. Assumption 1(b) implies that there exists a solution $w_{m-1}\in R^q$ of the equation (18).

Let ω_{m-1} be a number from Γ_{m-1}, and $w_{m-1}\in R^q$ be corresponding lexicographically minimal solution of the equation (18). Then for the solution $z_{m-1}(t)$, $\varepsilon_{m-1}\le t\le t_1+\theta_1$, with the initial condition $z_{m-1,0}=z_{m-1}(\varepsilon_{m-1})$, using (11) and letting $i=m-1$ we have the following:

for $t\in[\varepsilon_{m-1},\varepsilon_m]$,

$$F_{m-1,1}\pi_{m-1,1}z_{m-1}(t)=\phi_{m-1}(t,z_{m-1,0})-\frac{(t-\varepsilon_{m-1})^{k_{m-1}}}{k_{m-1}!}[\omega_{m-1}-k_{m-1}!\cdot(t-\varepsilon_{m-1})^{-k_{m-1}}h_{m-1}(t,\varepsilon_{m-1})]$$

$$F_{m-1,2}\pi_{m-1,2}z_{m-1}(t)=g_{m-1}(t,z_{m-1,0});\qquad (19)$$

and for $t\in[\varepsilon_m,t_1+\theta_1]$,

$$F_{m-1,1}\pi_{m-1,1}z_{m-1}(t)=\phi_{m-1}(t,z_{m-1,0})-\frac{(t-\varepsilon_{m-1})^{k_{m-1}}}{k_{m-1}!}[\omega_{m-1}-\omega_{m-1}\left(\frac{t-\varepsilon_m}{t-\varepsilon_{m-1}}\right)^{k_{m-1}}-\gamma_{m-1}(t)\cdot\frac{(t-\varepsilon_m)^{k_{m-1}-\frac{1}{2}}}{(t-\varepsilon_{m-1})^{k_{m-1}}}$$
$$-k_{m-1}!\cdot(t-\varepsilon_{m-1})^{-k_{m-1}}h_{m-1}(t,\varepsilon_m)],$$

$$F_{m-1,2}\pi_{m-1,2}z_{m-1}(t)=g_{m-1}(t,z_{m-1,0}).\qquad (20)$$

where

$$\gamma_{m-1}(t)=\int_{\varepsilon_m}^{t}\frac{(t-\tau)^{k_{m-1}-1}}{(k_{m-1}-1)!}F_{m-1,1}\pi_{m-1,1}C_{m-1}^{k_{m-1}-1}\left[D_{m-1}v(\tau)-B_{m-1}u(\tau)\right]d\tau\cdot(t-\varepsilon_m)^{-k_{m-1}+\frac{1}{2}}.$$

On the interval $\varepsilon_m\le t\le t_1+\theta_1$, we set $v(t)=v_m(t)=G_mu_m(t)+w_m$, where $w_m\in R^q$ is a solution of the equation

$$F_{m1}\pi_{m1}C_m^{k_m-1}D_mw_m=-\omega_m \qquad (21)$$

where $\omega_m\in\Gamma_m=\left\{\omega:|\omega|\le b_m,b_m>0\right\}$. From Assumption 1(b) implies that there exists a solution $w_m\in R^q$ of the equation (21). Then for the solution $z_m(t)$, $\varepsilon_m\le t\le t_1+\theta_1$, with $z_{m0}=z_m(\varepsilon_m)$, we obtain from (11) at $i=m$ the following equation

$$F_{m1}\pi_{m1}z_m(t)=\phi_m(t,z_{m0})-\frac{(t-\varepsilon_m)^{k_m}}{k_m!}\left[\omega_m-k_m!\cdot(t-\varepsilon_m)^{-k_m}h_m(t,\varepsilon_m)\right]$$

$$F_{m2}\pi_{m2}z_m(t)=g_m(t,z_{m0}),\ \varepsilon_m\le t\le t_1+\theta_1 \quad (22)$$

Using the Cauchy–Schwartz inequality and admissibility of controls of players we can see that there exist a positive numbers d_i^1 such that

$$|\gamma_i(t)|\le d_i^1,\ \varepsilon_{i+1}\le t\le t_1+\theta_1,\ i\in\{1,2,\dots,m-1\}. \quad (23)$$

Next, consider the vector-function $\psi_i(t,z_{i0})=(k_i!(t-\varepsilon_i)^{-k_i}\phi_i(t,z_{i0}),g_i(t,z_{i0}))$. Obviously, it has at most one solution on the interval $[\varepsilon_i,t_1+\theta_1]$. It is clear that on the segment Γ_i it has no more then one zero. For this reason , if we divide the segment Γ_i into two equal parts Γ_i^1, Γ_i^2, then the curve $\psi_i(t,z_{i0})$ doesn't intersect at least one of them, without loss of generality, we assume that it is Γ_i^1, and let ω_i^0 be its midpoint. Choose the numbers $\varepsilon_1<\varepsilon_2<\ldots<\varepsilon_m<t_1$ such that

$$\left|\omega_i^0\cdot\left(\frac{t_1-\varepsilon_{i+1}}{t_1-\varepsilon_i}\right)^{k_i}-d_i^1\frac{(t_1-\varepsilon_{i+1})^{k_i-\frac{1}{2}}}{(t_1-\varepsilon_i)^{k_i}}-d_i(t_1-\varepsilon_i)^{\frac{1}{2}}\right|<\frac{b_i}{4},$$
$$i\in\{1,2,\dots,m-1\},\qquad (24)$$

$$\left|d_i(t_1-\varepsilon_i)^{\frac{1}{2}}\right|<\frac{b_i}{4},\ i\in\{1,2,\dots,m\}.\qquad (25)$$

Since b_i, d_i, d_i^1, $i\in\{1,2,\dots,m\}$ are positive constants, therefore we are able to choose numbers satisfying the inequalities (24) and (25). Note that the numbers $\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_m$ are found consecutively: first $\varepsilon_1=t_1-\theta_1\ge0$ is chosen. Then ε_2 is found based on

ε_1 and so on. Assume that all the numbers $\varepsilon_1, \varepsilon_2, \varepsilon_3, ..., \varepsilon_m$ has been chosen. In view of (10), the functions $h_i(t, \varepsilon_i)$ satisfy the inequality

$$\left|h_i\left(t, \varepsilon_i\right)\right| \le \frac{d_i}{k_i!}(t-\varepsilon_i)^{k_i+1/2},\ \varepsilon_i \le t \le t_1 + \theta_1 .$$

Then combining (23), (24) and (25), gives

$$w_i(t_1)=\omega_i^0-\omega_i^0\left(\frac{t_1-\varepsilon_{i+1}}{t_1-\varepsilon_i}\right)^{k_i}-\gamma_i(t_1)\cdot\frac{(t_1-\varepsilon_{i+1})^{k_i-\frac{1}{2}}}{(t_1-\varepsilon_i)^{k_i}}-k_i!(t_1-\varepsilon_i)^{-k_i}h_i(t_1,\varepsilon_i)\in\Gamma_i^1,$$
$$i\in\{1,2,\dots,m-1\},$$
$$w_i^*(t_1)=\omega_i^0-k_i!(t_1-\varepsilon_i)^{-k_i}h_i(t_1,\varepsilon_i)\in\Gamma_i^1,$$
$$i\in\{1,2,\dots,m\}.$$

Taking $\omega_1=\omega_1^0$, $\omega_2=\omega_2^0$, $\omega_{m-1}=\omega_{m-1}^0$, $\omega_m=\omega_m^0$ in the equations (13)-(14), (16)-(17), (19)-(20), and (22) respectively, we get the following inequalities at $t=t_1$

$$(k_i!(t_1-\varepsilon_i)^{-k_i}F_{i1}\pi_{i1}z_i(t_1),F_{i2}\pi_{i2}z_i(t_1))=$$
$$(k_i!(t_1-\varepsilon_i)^{-k_i}\varphi_i(t_1,z_{i0}),g_i(t_1,z_{i0}))-$$
$$-(w_i(t_1),0)\neq 0,$$
$$i\in\{1,2,\dots,m\}.$$

or

$$(k_i!(t_1-\varepsilon_i)^{-k_i}F_{i1}\pi_{i1}z_i(t_1),F_{i2}\pi_{i2}z_i(t_1))=$$
$$(k_i!(t_1-\varepsilon_i)^{-k_i}\varphi_i(t_1,z_{i0}),g_i(t_1,z_{i0}))-(w_i^*(t_1),0)\neq 0,$$
$i\in\{1,2,\dots,m\}$. This implies that

$$(F_{i1}\pi_{i1}z_i(t_1),F_{i2}\pi_{i2}z_i(t_1))\neq 0,\ i\in\{1,2,\dots,m\}. \quad (26)$$

Since maps F_{ij} are linear, and one to one, π_{ij} is orthogonal projection operator of R^{n_i} onto W_{ij} and $g_i(t,z_{i0})\neq 0$ for all $t\neq t_1$, $t\in I_0\cup I_1$, therefore (26) implies that $z_i(t)\notin M_i$, $t\in I_0\cup I_1$, $i\in\{1,2,...,m\}$. Thus, we have proved that evasion is possible on the interval $I_0\cup I_1$. The same proof works for the intervals $I_2, I_3,\dots,I_k,\dots$. Note that $I_j\cap I_k=\emptyset$, $j\neq k$. Therefore, we set $v(t)=0$ outside the intervals I_j, $j=1,2,3,\dots$. This completes the proof that evasion is possible in the game (1) -(3).

It remains to prove that the control $v(t)$, $0\le t<\infty$ is admissible. By Assumption 1 (a), there exists a compact set $Q_1\subset R^q$ such that

$$\Gamma_i\subset F_{i1}P_{i1}C_i^{k_i-1}D_iQ_1,\ i\in\{1,2,\dots,m\}.$$

Set $\delta=\sigma^2-\|G_1\|^2\rho_1^{\,2}-\ldots-\|G_m\|^2\rho_m^{\,2}$,

$$w_0=\max_{w\in Q_1}|w|^2,$$

$$\theta_j^*=\min\left\{\frac{1}{2},\ \theta_j,\ \frac{\delta^2}{2^{j+1}w_0^2\left(mw_0+2\|G_1\|\rho_1+\ldots+2\|G_m\|\rho_m\right)^2}\right\}$$

Then using the integral inequality of Minkowski, we obtain the following estimate

$$\int_0^\infty|v(t)|^2\,dt=\sum_{j=1}^\infty\int_{t_j-\theta_j^*}^{t_j+\theta_j^*}|v(t)|^2\,dt$$

$$=\sum_{j=1}^\infty\left(\int_{\varepsilon_1}^{\varepsilon_2}|G_1u_1(t)+w_1|^2dt+\int_{\varepsilon_2}^{\varepsilon_3}|G_2u_2(t)+w_2|^2dt+\ldots+\int_{\varepsilon_m}^{t_j+\theta_j^*}|G_mu_m(t)+w_m|^2dt\right)$$

$$\le\|G_1\|^2\rho_1^2+\|G_2\|^2\rho_2^2+\ldots+\|G_m\|^2\rho_m^2+w_0\left(mw_0+2\sum_{i=1}^m\|G_i\|\rho_i\right)\sqrt{\sum_{j=1}^\infty\theta_j^*}\le\sigma^2$$

and the proof of Theorem 1 is complete.

Theorem 2. Let Assumptions1,3 be satisfied and Let the initial point $z_0=(z_{10},z_{20},\cdots,z_{m0})$ be such that $z_{i0}\notin M_i$, $\Phi_i(z_{i0})\neq 0$ for all $i\in\{1,2,...,m\}$. Then evasion is possible from the point $z_0=(z_{10},z_{20},\cdots,z_{m0})$ in the game (1)-(3).

Proof. By the hypothesis of the theorem $\Phi_i(z_{i0})\neq 0$, $i\in\{1,2,...,m\}$. Comparing summands of the function $\Phi_i(z_{i0})$ with the system of equations (8) and coefficients of the functions $g_1(t,z_{10}),...,\ g_m(t,z_{m0})$, we can see that for the given point z_0 the system of equations (8) inconsistent and all the functions $g_1(t,z_{10}),...,g_m(t,z_{m0})$ are not identically equal to zero on $[0,\infty)$. The rest of the proof runs as the proof of Theorem 1.

4. Extension of obtained results

Consider a differential game in R^{n_i} described by the following equations

$$\dot{z}_i = C_i z_i - B_i U_i + D_i V + a_i \qquad (27)$$

where C_i, B_i, D_i are constant matrices with appropriate dimensions, $U_i = \begin{pmatrix} u_i \\ u_i^0 \end{pmatrix}$, $V_i = \begin{pmatrix} v \\ v^0 \end{pmatrix}$, $u_i^0 \in R^{p_{2i}}$, $v^0 \in R^{q_2}$ are constant uncontrolled vectors, $u_i \in R^{p_{1i}}$, $v \in R^{q_1}$ are control parameters of the i-th pursuer and evader, respectively, subjected to constraints

$$\|u_i(\cdot)\|_{L_2} \le \rho_i, \quad (28)$$

$$\|v(\cdot)\|_{L_2} \le \sigma. \quad (29)$$

To formulate the main results, we give assumptions.

Assumption 4. There exist positive integers k_i, one-dimensional subspaces W_{i1}, W_{i2}, and linear one to one maps $F_{ij} : W_{ij} \to R^1$, $j = 1,2$, such that

(a) each of the sets

$$F_{i2}\pi_{i2}C_i^{\ell j}\left(B_i\begin{pmatrix} R^{p1i} \\ u_i^0 \end{pmatrix} + D_i\begin{pmatrix} R^{q1} \\ v^0 \end{pmatrix}\right),\ i \in \{1,2,\dots,m\},$$

$$\ell j = 0,1,\dots,k_{ij}-2, k_{i1} = k_i, k_{i2} = n_i - r_i + 1,$$

is singleton and $C_i^{r_i} \ne 0$, where r_i is multiplicity of zero solution of characteristic equation of matrix C_i;

(b) $$F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} R^{q_1} \\ v^0 \end{pmatrix} = F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} R^{q_1} \\ 0 \end{pmatrix} = R^1,\ \forall i \in \{1,2,\dots,m\};$$

(c) $$F_{i1}\pi_{i1}C_i^{k_i-1}B_i\begin{pmatrix} R^{p_{1i}} \\ u_i^0 \end{pmatrix} = F_{i1}\pi_{i1}C_i^{k_i-1}B_i\begin{pmatrix} R^{p_{1i}} \\ 0 \end{pmatrix},$$

$\forall i \in \{1,2,\dots,m\}$

By the Hamilton – Kayley theorem, any matrix satisfies its characteristic equation. Therefore, Assumption 4(a) implies that the set $F_{ij}\pi_{ij}C_i^r\left(B_i\begin{pmatrix} R^{p_{1i}} \\ u_i^0 \end{pmatrix} + D_i\begin{pmatrix} R^{q_1} \\ v^0 \end{pmatrix}\right)$ is singleton for all $r \ge n_i - r_i$.

Set

$$a_{ij} = F_{i2}\pi_{i2}C_i^j a_i + F_{i2}\pi_{i2}C_i^j\left(-B_i\begin{pmatrix} R^{p_{1i}} \\ u_i^0 \end{pmatrix} + D_i\begin{pmatrix} R^{q_1} \\ v^0 \end{pmatrix}\right),$$

$$\forall i \in \{1,2,\dots,m\}\ j = 1,2,\dots,$$

$$\Phi_i(z_i) = \sum_{j=0}^{n_i}(F_{i2}\pi_{i2}C_i^j z_i - a_{ij-1})^2, a_{i,-1} = 0,$$

$$i \in \{1,2,\dots,m\}.$$

Note that in this case, in contrast to Assumption 1, some of the numbers a_{ij} might not equal to zero. Define matrices H_i, H_i^* similar to (7).

Assumption 5. $rankH_i < rankH_i^*$ for all $i \in \{1,2,\dots,m\}$.

Observe that by Assumption 4 (b)

$$F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} R^{q_1} \\ v^0 \end{pmatrix} = F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} R^{q_1} \\ 0 \end{pmatrix} = R^1,$$

$\forall i \in \{1,2,\dots,m\}$.

Therefore, using Assumption 4(c), we can assert that there are linear maps $G_{1i} : R^{p_{1i}} \to R^{q_1}$ such that

$$F_{i1}\pi_{i1}C_i^{k_i-1}B_i\begin{pmatrix} E_i \\ u_i^0 \end{pmatrix} = F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} G_{1i} \\ v^0 \end{pmatrix},$$

$\forall i \in \{1,2,\dots,m\}$.

Let

$$\alpha_{1i} = \inf\left\{\|G_{1i}\| \,\Big|\, F_{i1}\pi_{i1}C_i^{k_i-1}B_i\begin{pmatrix} E_i \\ u_i^0 \end{pmatrix} = F_{i1}\pi_{i1}C_i^{k_i-1}D_i\begin{pmatrix} G_{1i} \\ v^0 \end{pmatrix}\right\}$$

Assumption 6.

$$\sigma^2 > \alpha_{11}^2\rho_1^2 + \alpha_{12}^2\rho_2^2 + \dots + \alpha_{1m}^2\rho_m^2.$$

Theorem 3. Let Assumptions 4–6 hold. Then evasion is possible in the game with integral constraints (26) -(28).

Theorem 4. Let Assumptions 4 and 6 hold. If $\Phi_i(z_{i0}) \ne 0$ for the points $z_{i0} \notin M_i$, $i \in \{1,2,\dots,m\}$, then evasion is possible in the game (26)-(28) from the point $z_0 = (z_{10}, z_{20}, \dots, z_{m0})$.

Theorems 3 and 4 can be proved by the same scheme as Theorems 1 and 2 with small changes.

Remark. Conclusions of these theorems are still valid if control functions of players $u_i = u_i(\cdot), v = v(\cdot)$ satisfy general integral constraints: $\|u_i(\cdot)\|_{Lp} \le \rho_i$, $\|v(\cdot)\|_{Lq} \le \sigma$.

Example. As a model example, we consider the Pontryagin example, where movements of the pursuers are described by equations

$$\ddot{x}_i + \alpha_i \dot{x}_i = U_i + g_i \ , \qquad (30)$$

and movement of the evader is described by the equation

$$\ddot{y} + \beta\dot{y} = V + h \ , \qquad (31)$$

where $x_i,\ y,\ U_i, V \in R^n,\ n \ge 2$,

$$U_i = \begin{pmatrix} u_i \\ u_i^0 \end{pmatrix},\ V_i = \begin{pmatrix} v \\ v^0 \end{pmatrix},\ u_i^0 \in R^{p_{2i}}\ ,\ v^0 \in R^{q_2}$$

uncontrolled vectors, $u_i \in R^{p_{1i}}$, $v \in R^{q_1}$ are control parameters of the i-th pursuer, $i \in \{1,2,...,m\}$, and evader, respectively, which are subjected to constraints (28) and (29), respectively, $\alpha_i, \beta, \rho_i, \sigma$ are given non negative numbers. Clearly, $p_{1i} + p_{2i} = q_1 + q_2 = n$.

We say that evasion is possible if $x_i(t) \ne y(t)$ at some $t > 0$ and for all $i \in \{1,2,...,m\}$. Using reduced coordinates $z_i = (z_{1i}, z_{2i}, z_3)$ defined by $y - x_i = z_{1i},\ \dot{x}_i = z_{2i},\ \dot{y} = z_3$, we can rewrite (30) and (31) as

$$\dot{z}_i = C_i z_i - B_i U_i + D_i V + l_i \qquad (32)$$

where

$$C_i = \begin{bmatrix} 0 & -E & E \\ 0 & -\alpha_i E & 0 \\ 0 & 0 & -\beta E \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ E \\ 0 \end{bmatrix},\ D_i = \begin{bmatrix} 0 \\ 0 \\ E \end{bmatrix},\ l_i = \begin{bmatrix} 0 \\ g_i \\ h \end{bmatrix},$$

E is $n \times n$ identity matrix.

In reduced coordinates the terminal set has the form $M_i = \{(z_{1i}, z_{2i}, z_3):\ z_{1i} = 0\}$. Then $L_i = \{(z_{1i}, z_{2i}, z_3): z_{2i} = 0,\ z_3 = 0\}$ is the orthogonal complement of M_i in R^{3n}, and

$W_{i1} = \{(z_{1i}, z_{2i}, z_3): [z_{1i}]_1 = 0,\ z_{2i} = 0,\ z_3 = 0\} \subset L_i$,

$W_{i2} = \{(z_{1i}, z_{2i}, z_3): [z_{1i}]_2 = 0,\ z_{2i} = 0,\ z_3 = 0\} \subset L_i$,

where, by $[f]_s$ denotes the s-th coordinate of the vector f ($s = 1,2$).

For simplicity, we set $n = 2$ end application of Theorem 3 to the game (32). Clearly, $p_{1i} = p_{2i} = q_1 = q_2 = 1$, for the game (32) we can calculate matrices

$$C_i^j = \begin{bmatrix} 0 & (-1)^j \alpha_i^{j-1} E_i & (-1)^{j+1} \beta^{j-1} E_i \\ 0 & (-\alpha_i)^j E_i & 0 \\ 0 & 0 & (-\beta)^j E_i \end{bmatrix},\ j \ge 2,$$

$r_i = 1$, and $k_{i1} = k_i = 2$, $k_{i2} = 4$ is holds all conditions of the Assumption 4. Define numbers $a_{ij} = (-\alpha_i)^j u_i^0 + (-1)^{j+1} \beta^j v^0$ and matrices H_i, H_i^* similar to (7). Therefore, at least one of the following conditions holds

(i) $\alpha_i = \beta,\ u_i^0 \ne v^0,\ i \in \{1,2,...,m\}$; (ii) $\alpha_i \ne \beta,\ \alpha_i u_i^0 \ne \beta v^0,\ i \in \{1,2,...,m\}$,

then is hold Assumption 5 for the game (32). Further, if $\sigma^2 > \rho_1^2 + \rho_2^2 + \cdots + \rho_m^2$ then is hold true Assumption 6 for the game (32). Therefore in the game (32) holds true all Assumptions 4-6 and according to the theorem 3 evasion is possible in the game (32).

If both of the conditions (i) and (ii) fails to hold, the none cannot apply Theorem 3 to the game (32). However, in this case, theorem 4 is applicable to the game (32) and it leads to the following result: if $\sigma^2 > \rho_1^2 + \rho_2^2 + \cdots + \rho_m^2$, and for the numbers $z_{i0} \notin M_i$

$$\Phi_i(z_{i0}) = ([z_{1i}^0]_2)^2 + ([z_{2i}^0 + z_3^0]_2)^2 +$$
$$+ ([-\alpha_i z_{2i}^0 + \beta z_3^0 + u_i^0 - v^0]_2)^2 +$$
$$+ ([\alpha_i^2 z_{2i}^0 - \beta^2 z_3^0 - \alpha_i u_i^0 - \beta v^0]_2)^2 \ne 0$$

for all $i \in \{1,2,...,m\}$, then evasion is possible in the game (32) only from the point $z_0 = (z_{10}, z_{20}, \cdots, z_{m0})$.

Conclusion

In the present paper, we have obtained new sufficient conditions of evasion from many pursuers in critical case when the projection of evader's control set is a segment parallel to coordinate axes. A new method has been proposed to control the evasion parameter, which allows simultaneously avoiding from several pursuers and requires a minimum advantage of resource of evader over the resources of pursuers. The proposed method can be applied for solution of problems of the theory of pursuit and evasion differential games as well as problems of mathematical control theory, when controls are subjected to integral constraints.

References

1. AzimovA.Ya. (1974). Linear evasion differential game with integral constraints on controls. Journal of Computational Mathematics and Mathematical Physics. 14(6), 1427-1436.

2. Chikrii A.A. (1984) Method of alternating directions in nonlinear differential games of evasion. Kibernetika. No 1, pp.48–54.

3. Gusiatnikov P.B. (1975) Escape of nonlinear objects of different types with integral constraints on the control.PMM, Vol. 39, No 1, 1975, pp. 12-23

4. Kuchkarov A.S., Rikhsiyev B.B. (2002). The possibility of avoiding an encounter in a linear differential game of evasion. Journal of Applied Mathematics and Mechanics, 66(2), 233–237. doi:10.1016/S0021-8928(02)00028-X

5. Mesencev A.V. (1974). Sufficient escape conditions for linear games with integral constraints. Dokl. Akad. Nauk SSSR, 218(5) 1021-1023.

6. Mishchenko E.F. (1974). On the problem of evading the encounter in differential games.SIAM J. Control, 12(2), 300–310.

7. Mishchenko E.F., Nikol'skii M.S., Satimov N.Y. (1977). Evoidance encounter problem in differential games of many persons. Trudy MIAN USSR,143, 105–128.

8. Mishchenko E.F., Satimov N.Y. (1981). On the possibility of evading encounter in differential games in a critical case. Proceedings of the Steklov Institute of Mathematics, 158, 131–134.

9. Mishchenko E.F., Satimov N.Y. (1983)An Encounter–Evasion Problem in the Critical Case. Differential Equations, 19(2), 160–167

10. Nikolskii M.S. (1974) On a linear escape problem. Dokl. Akad. Nauk SSSR, 218, pp. 1024-1027. (In Russian) Soviet Math. Dokl., 15 (1974), pp. 1462-1466. (In English.)

11. Ostapenko V.V. (1978) A nonlinear escape problem. Kibernetika (Kiev), 14, pp. 106-112. (In Russian.) Cybernetics, 14 (1978), pp. 594-601. (In English.)

12. Pontryagin L.S., Mishchenko E.F. (1971) The problem of evading the encounter in linear differential games. Differentsialnye Uravneniya. 7, pp. 436-445.

13. Pontryagin L.S. (1971). Linear evasion differential game. Proceedings of the Steklov Institute of Mathematics, 112: 30–63.

14. Pshenichnii B.N. (1968) Linear differential games. Avtomafika and Telemekhanika. 1, 65-78.

15. Rakhmanov A.T. (1984) On a problem of evasion from many pursuers. Dokl. Akad. UzSSR, No 8, p.3-4.

16. Rakhmanov A.T. (1985) Sufficient conditions of positional evasion in the critical case. Izv. Akad. NaukUzSSR, Ser. Fiz.-Mat. Nauk, No.6, pp.21–26.

17. Rakhmanov A.T. (1987) Linear evasion differential games with integral constraints in the critical case.Izv. Akad. NaukUzSSR, Ser. Fiz.-Mat. Nauk, No.1, pp.34–40.

18. Satimov N.Yu. (1984). Possibility of Encounter Avoidance in a Critical Case. Differential Equations, 20:12 1451–1455.

19. Satimov N.Yu., Rikhsiev B.B. (1978) Quasilinear differential games of evasion. Diferents. Uravneniya, 14(6): 1046-1052.

20. Satimov N. Yu., Rikhsiev B. B. (2000). Methods for Solving the Evasion Problem in Mathematical Control Theory. Tashkent, Fan.

21. Yugai L.P. (1996) A sufficient condition for course evasion. Differential Equations. 32:9, 1294–1296

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АХБОРОТ БИРЛИКЛАРИГА БЎЛИШ АСОСИДА ПРОДУКЦИОН БИЛИМЛАР БАЗАСИНИ ЛОЙИҲЛАШ ЁНДАШУВИ

Мақолада электрон ахборот ресурсларида билимларни акс эттиришда танланган предмет соҳа бўйича ахборот бирликларига бўлиш асосида продукцион билимлар базасини лойиҳалаш ёндашуви қараб ўтилган. Бундай ёндашув сирасига предмет соҳанинг турли хусусиятларини шакллантириш ҳамда билимларни акс эттиришда сабаб-моҳият кўринишидаги боғланишлар амалга оширилиши келтирилган. Бунда танланган предмет соҳани глобал ва локал ахборотлар сифатида ажратиб олиш ёрдамида предмет соҳанинг сабаб-моҳият кўринишдаги боғлиқликлар асосида билимларни ифодаланиши асосланган. Продукцион модел ёрдамида билимлар базасидан билимларни қидириб топиш ёндашувлари келтирилган.