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Methods of protection of national security from external and internal information threats

The article is devoted to classification of possible external and internal information threats and computer crimes, the arising problems: the prompt strengthening of processes of a computerization of society, the law enforcement agencies putting unawares which were not ready to adequate opposition in fight against this new social phenomenon and also development of complex measures for ensuring national security.
Keywords: categorization, external, internal, information threats, provision of national safety.

UDC 004.942

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TRANSITION RULES OF LINEAR CELLULAR AUTOMATA OVER BINARY FIELD AND IMAGE ANALYSIS

This paper investigates the theoretical aspects of two-dimensional (2D) linear cellular automata (CA) over binary field combining with image science application. The present study focuses on the theory of 2D linear CA with respect to uniform null and adiabatic boundary CA conditions. It is considered geometrical and visual aspects of patterns generated by these CA evolution. Multiple copies of any arbitrary seed image corresponding to CA could be obtained by using the transition rules of these CAs. Also we believe that these type of CA could be found many different applications in special case situation e.g. computability theory, mathematics, theoretical biology, DNA genetics research, image science, textile design, video processing and microstructure modeling., etc. in near future.
Keywords: Cellular automata, null and adiabatic boundary, CA and Image analisys.

Introduction

Cellular automata (CAs for brevity) introduced by Ulam and von Neumann [1] in the early 1950’s, have been systematically studied by Hedlund from purely mathematical point of view. One-dimensional CA has been investigated to a large extent. However, little interest has been given to two-dimensional cellular automata (2D CA). Von Neumann [1] showed that a cellular automaton can be universal. Due to its complexity, von Neumann rules were never implemented on a computer. In the beginning of the eighties, Wolfram [2] has studied in much detail a family of simple one-dimensional (1D) CA rules and showed that even these simplest rules are capable of emulating complex behavior. Some basic and precise mathematical models using matrix algebra over the binary field which characterize the behavior of 2D nearest neighborhood linear CA with null and periodic boundary conditions have been seen in the literature [3, 7, 8, 9, 10, 11]. CA has received remarkable attention in the last few decades [11, 12, 13, 14, 15, 16]. Due to its structure CA has given the opportunity to model and understand many behaviors in nature easier. Most of the work for CA is done for one-dimensional (1D) case. The paper [14] deals with the behavior of the uniform 2D CA over binary fields.

Here we study the theory of 2-dimensional uniform null and adiabatic boundary CA (2D NB CA, 2D AB CA) of the all linear rules (e.g. von Neumann, Moore neighborhood and the others) and applications of image processing for self replicating patterns (see Figs. 4-6). We present some illustrative examples and figures to explain the method in detail. Using the rule matrices obtained in this work, the present paper contributes further to the algebraic structure of these CA and relates its applications studied by different authors previously (i.e. [4, 5, 6]).

In this paper, we concentrate a special family of 2D finite linear CA with null and adiabatic boundary condition over the binary field Z_2. Here, we set up a specific relation between the structure of these CA and transition matrix rules of 2D linear CA with null and adiabatic boundary condition. It is determined of the transition rule matrices of this special CA by means of the matrix algebra theory. Due to CA nature is very simple to allow some important mathematical studies and to obtain very complicated and complex behaviors chaos in dynamical systems, it is believed that these linear CA construction could be obtained many different kind of applications. Using the linear rule matrices presented in the work, the present results give further to the algebraic consequences of these 2D CA and relates some elegant applications found by the authors in the literature (i.e. [11, 12, 15, 16, 17]).

Preliminaries

In this paper, we work CA established by rules over the field Z_2, and deal with the general case of the transition rule matrices.

Technical Details and Rule Matrix

In this section, we introduce 2D CA over the field Z_2 by using all primary local rules. We recall the definition of a CA. We consider the 2-dimensional integer lattice Z^2 and the configuration space $W = \{0,1\}^{Z^2}$ with elements $\sigma: Z^2 \to \{0,1\}$. (see details in [16, 17]).

The 2D finite CA consists of $m \times n$ cells arranged in m rows and n columns, where each cell takes one of the values of 0 or 1. A configuration of the system is an assignment of states to all the cells. Every configuration determines a next configuration via a linear transition rule that is local in the sense that the state of a cell at time $(t + 1)$ depends only on the states of some of its neighbors at time t using modulo 2. For 2D CA nearest neighbors (see Figs. 1-3), there are nine cells arranged in a 3 × 3 matrix centering that particular cell (see [S6, S9, S10] for the details). For 2D CA there are some classic types of rules, but in this work only we restrict ourselves to primary linear rules (briefly, from Rule 1 to Rule 256). So, we can define (for an example Rule 170) the $(t+1)^{th}$ state of the $(i,j)^{th}$ cell as follows;

$$x_{(i,j)}^{(t+1)} = x_{(i-1,j)}^{(t)} + x_{(i,j+1)}^{(t)} + x_{(i+1,j)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod 2 \qquad (\textit{Rule}170)$$

Any other rules can be written the linear combination of primary rules of Fig. 2. The dependence will be restricted to the case of being zero or nonzero, in other words if the coefficients in the above equation equal to 0 or 1, then this case will be assumed to be the same. This approach is adopted in this paper though these cases may be further distinguished. The linear combination of the neighboring cells on which each cell value is dependent is called the rule number of the 2D CA over the field Z^2.

In this paper, we will only consider a 2D finite CA generated by the primary rules with null boundary (NB) and adiabatic boundary (AB). It is well known that these CAs are discrete dynamical systems formed by a finite 2D array $m \times n$ composed by identical objects called cells.

Let $F : M_{mn}(Z_2) \to Z_2^{mn}$. F takes the t^{th} state $[X_t]$ given by

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & \cdots & x_{1n} \\ x_{21} & x_{22} & \vdots & \vdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & \cdots & x_{mn} \end{pmatrix} = (x_{11} \quad x_{12} \quad \cdots \quad x_{1n} \quad \cdots \quad x_{m1} \quad \cdots \quad x_{mn})^T,$$

where T is the transpose of the matrix. Therefore, the local rules will be assumed to act on Z_2^{mn} rather than $F : M_{m\times n}(Z_2)$. Suppose binary information matrix is $[X_t]_{m\times n}$, of order

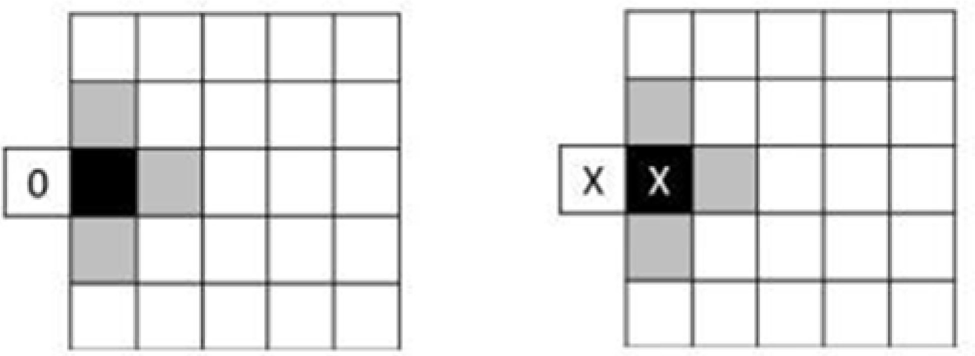

Fig.1: Null and adiabatic boundary configurations on 2D CA respectively

$m \times n$:

$$[X_t]_{m\times n} = \begin{pmatrix} x_{11}^{(t)} & \cdots & \cdots & x_{1n}^{(t)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}^{(t)} & \cdots & \cdots & x_{mn}^{(t)} \end{pmatrix}$$

is called the configuration of the 2-D finite CA at time t. From the above equations, we can define as follows;

$$(T_{Rule})_{mn\times mn} \begin{pmatrix} x_{11}^{(t)} \\ \vdots \\ x_{1n}^{(t)} \\ \cdot \\ \cdot \\ x_{m1}^{(t)} \\ \cdot \\ \cdot \\ x_{mn}^{(t)} \end{pmatrix} = \begin{pmatrix} x_{11}^{(t+1)} \\ \vdots \\ x_{1n}^{(t+1)} \\ \cdot \\ \cdot \\ x_{m1}^{(t+1)} \\ \cdot \\ \cdot \\ x_{mn}^{(t+1)} \end{pmatrix}.$$

The matrix $(T_{Rule})_{mn\times mn}$ is called the rule matrix with respect to the 2D finite $CA_{m\times n}$ with the transition rule (see [6] for details).

Here we give some background of 2D CA boundary conditions. Considering the neighborhood of the information cells, there are two well known studied boundary approaches in the literature.

· Null or fixed boundary: A null boundary (NB) CA is the one whose extreme cells are connected to 0-state. (see Fig. 1)

	Rule 64	Rule 128	Rule 256	
	Rule 32	Rule 1	Rule 2	
	Rule 16	Rule 8	Rule 4	

Fig.2: Rule convention chart of 2D linear CA over binary field. Any other rules can be written the linear combination of these primary rules

· Adiabatic boundary: An adiabatic boundary (AB) CA is duplicating the value of the cell in an extra virtual neighbor. (see Fig. 1)

The linear combination of the neighboring cells on which each cell value is dependent is called the rule number of the 2D CA over the binary field Z_2.

Primary Transition Rule Matrices

In this section, we will obtain all primary transition rule matrices corresponding to 2D linear, null and adiabatic boundary CA over the binary field Z_2.

Transition Rule Matrices under Null Boundary

Theorem (Null Case). The transition rule matrices of 2D linear CA of all primary rules (1, 2, 4, 8, 16, 32, 64, 128, and 256) under the null boundary condition can be given in the following way:

$$T_{Rule1NB} = \begin{pmatrix} I & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & I & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & I & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & I & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & I \end{pmatrix}_{mn\times mn}$$

$$T_{Rule2NB} = \begin{pmatrix} A & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & A & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & A & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & A \end{pmatrix}_{mn\times mn}$$

$$T_{Rule4NB} = \begin{pmatrix} 0 & A & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & A \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule8NB} = \begin{pmatrix} 0 & I & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & I & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & I \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule16NB} = \begin{pmatrix} 0 & B & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & B & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & B & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & B \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule32NB} = \begin{pmatrix} B & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & B & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & B & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & B & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & B \end{pmatrix}_{mn\times mn}$$

$$T_{Rule64NB} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ B & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & B & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & B & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & B & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule128NB} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ I & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & I & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & I & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule256NB} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ A & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & A & 0 \end{pmatrix}_{mn\times mn}$$

where I is the identity $n \times n$ matrix and the sub-matrices A,B are given as

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{n\times n}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}_{n\times n}$$

	$x_{i-1,j-1}$	$x_{i-1,j}$	$x_{i-1,j+1}$	
	$x_{i,j-1}$	$x_{i,j}$	$x_{i,j+1}$	
	$x_{i+1,j-1}$	$x_{i+1,j}$	$x_{i+1,j+1}$	

Fig.3: Two dimensional 3 × 3 linear cellular automata with center$x_{i,j}$

Proof. In order to determine each rule matrix $T_{Rule\ NB}$ we need to determine the action of $T_{Rule\ NB}$ on the bases vectors. Firstly, we consider the linear transformation Ψ from $m \times n$ matrix space to itself. Next, we relate the transformation Ψ with rule matrix $T_{Rule\ NB}$. Let e_{ij} denote the matrix of size $m \times n$ where the (i, j) position is equal to one and the rest of the entries equal to zero.

It is well known that these vectors give the standard basis for this space (see [14]). Given e_{ij}, the image of e_{ij} which is $\Psi(e_{ij})$ is related to the transition rules neighbors. $\Psi(e_{ij})$ equals to a linear combination of its non-zero neighbors in the following way:

$$\psi(e_{ij}) = e_{i+1,j} + e_{i,j-1} + e_{i-1,j} + e_{i,j+1},$$

with a care on the bordering components of the matrix. Due to the neighboring relations that govern the rule, we obtain the rule matrices given in the theorem.

Corollary. The following matrix relations between the transitions rules are obtained as below.

$$(Rule\ 2N\ B)^T = Rule\ 32N\ B,$$
$$(Rule\ 4N\ B)^T = Rule\ 64N\ B,$$
$$(Rule\ 8N\ B)^T = Rule\ 128N\ B,$$
$$(Rule\ 16N\ B)^T = Rule\ 256N\ B$$

where T is the matrix transpose operator.

Proof. It is easily seen that $A^T = B$. The relations given in the corollary are found using this equality

Transition Rule Matrices under Adiabatic Boundary

Theorem (Adiabatic Case). The transition rule matrices of 2D linear CA of all primary rules (1, 2, 4, 8, 16, 32, 64, 128, and 256) under the adiabatic boundary condition can be given in the following way:

$$T_{Rule1AB} = \begin{pmatrix} I & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & I & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & I & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & I \end{pmatrix}_{mn\times mn}$$

$$T_{Rule2AB} = \begin{pmatrix} C & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & C & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & C & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & C & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & C \end{pmatrix}_{mn\times mn}$$

$$T_{Rule4AB} = \begin{pmatrix} 0 & C & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & C & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 & C \\ 0 & 0 & 0 & 0 & \dots & 0 & C \end{pmatrix}_{mn\times mn}$$

$$T_{Rule8AB} = \begin{pmatrix} 0 & I & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & I & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 & I \\ 0 & 0 & 0 & 0 & \dots & 0 & I \end{pmatrix}_{mn\times mn}$$

$$T_{Rule16AB} = \begin{pmatrix} 0 & D & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & D & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 & D \\ 0 & 0 & 0 & 0 & \dots & 0 & D \end{pmatrix}_{mn\times mn}$$

$$T_{Rule32NB} = \begin{pmatrix} D & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & D & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & D & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & D & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & D \end{pmatrix}_{mn\times mn}$$

$$T_{Rule64AB} = \begin{pmatrix} D & 0 & 0 & \dots & 0 & 0 & 0 \\ D & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & D & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & D & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & D & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule128AB} = \begin{pmatrix} I & 0 & 0 & \dots & 0 & 0 & 0 \\ I & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & I & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & I & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & I & 0 \end{pmatrix}_{mn\times mn}$$

$$T_{Rule256NB} = \begin{pmatrix} C & 0 & 0 & \dots & 0 & 0 & 0 \\ C & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & C & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & C & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & C & 0 \end{pmatrix}_{mn\times mn}$$

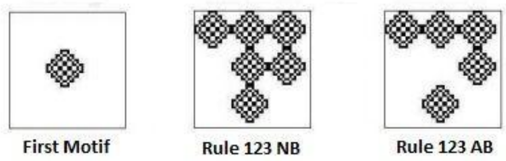

Fig. 4: Image application of Rule 123 with null and adiabatic boundaries after 16 iterations of the first seed image

where I is the identity $n \times n$ matrix and the sub-matrices C, D are given as

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}_{n\times n}$$

$$D = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}_{n\times n}$$

Proof. To determine each rule matrix $T_{Rule\ AB}$, it is needed to determine the action of $T_{Rule\ AB}$ on the bases vectors as given in the null case. Consider the linear transformation Ψ from $m \times n$ matrix space to itself. Next, we relate the transformation Ψ with rule matrix $T_{Rule\ AB}$. Given e_{ij}, the image of e_{ij} which is $\Psi(e_{ij})$ is related to the transition

rules neighbors. $\Psi(e_{ij})$ equals to a linear combination of its non-zero neighbors as in the null case. Then it is obtained the adiabatic transition rule matrices given in the theorem.

Image Analysis and Cellular Automata

Self replicating pattern generation is one of the most interesting topic and research area in nonlinear science. A motif is considered as a basic sub-pattern. Pattern generation is the process of transforming copies of the motif about the array (1D), plane (2D) or space (3D) in order to create the whole repeating pattern with no overlaps and blank [8, 9, 12, 13, 14, 15, 16]. These patterns have some mathematical properties which make generating algorithm possible. A cellular automaton is a good candidate algorithmic approach used for pattern generation.

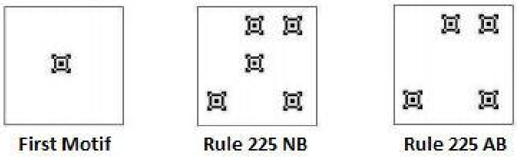

Fig. 5: Image application of Rule 225 with null and adiabatic boundaries after 16 iterations of the first seed image

Creating algorithmic approach for generating self replicating patterns of digital images (motif as in first image) is important and sometimes difficult task. Meanwhile many researchers face with many challenges in building and developing tiling algorithms such as providing simple and applicable algorithm to describe high complex patterns model. Growth from simple motif in 2D CAs can produce self replicating patterns with complicated boundaries (null, periodic, adiabatic and reflexive), characterized by a variety of growth dimensions. The approach given here leads to an accurate algorithm for generating different patterns. In this paper we use the CA with all the nearest neighborhoods (i.e. all primary rules and their combinations) to generate self replicate patterns of digital images. For applying 2D null, periodic and adiabatic CA linear rules in image processing, we take a binary matrix of size (100 × 100) due to computational limitations. We map each element of the matrix to a unique pixel on the screen (writing new MATLAB codes) and we color a pixel white for 0, black for 1 for the matrix elements. Then we take another image (as a motif) whose size is less than (30 × 30) for which patterns are to be generated and put it in the center of the binary matrix. This is the way how the image is drawn within an area of (100 × 100) pixels. It is observed from the figures that the self replicating patterns can be generated only when number of repetition is 2^n where $n = 4$ (i.e. 16-iterations give self replica images). A neighborhood function that specifies which of the adjacent cells affects its state also determines how many copies will be obtained from the self-replicating process. In the two dimensional and eight neighborhoods case, this should be at most eight copies of the original image itself (see Figs. 4-6). This situation brings also some limitation over the matrix size of the images to be replicated. The matrix size of the original images should lower 30 percent of the display matrix in all directions. If the first image exceeds 30 percentage of the length of row or column of the display matrix self replication pattern when

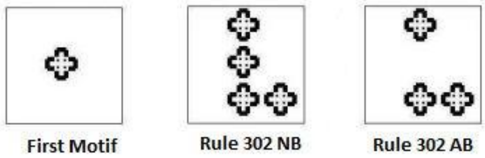

Fig.6: Image application of Rule 302 with null and adiabatic boundaries after 16 iterations of the first seed image

the iteration number t reaches to 16 does not occur. Also behaviors for different boundaries produce different shapes when t = 16. These results should be investigated in the next studies.

Conclusion

In this paper we present the theory two dimensional, uniform null and adiabatic boundary CA of linear primary rules and application of image science area. It can be seen that 2D linear CA theory can be applied successfully in self replica patterns of image science. The some characterization and applications on a 2D finite linear CA by using matrix algebra built on ternary field should be investigated for the next studies. However after making use of the matrix representation of 2D CA, it will be provided an algorithm to obtain the number of Garden of Eden configurations for the 2D CA defined by some rules. Some other interes-ting results and further connections on this direction wait to be explored in 2D CA and the other science branches, see Refs. [9-14].

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