

$$Q_k = \|q_{ij}\| - \text{cost matrix between the node;}$$

$$H_k = \|h_{ij}\| - \text{time distribution function for the formation and disbanding of compounds;}$$

$$h_k = \|h_i\| - \text{time vector for formation and disintegration of formulations;}$$

$$p^r - \text{number of program runs.}$$

Let's consider the work of the model on the first run ($i = 1$). The software takes into account that in the network, in addition to transit flows, there are internal transport flows. These internal threads affect the network bandwidth. Thus, on each run the capacity matrix $c [i, j]$ is changed by playing a single lot of the third type.

Based on the application of the Ford-Falkerson algorithm, the maximum flux maxflow, the distribution of the flow in the network $f [i, j]$, is calculated, and the generalized indicator of the "benefit" of this flux Φ_{zy} is determined by formulas (2), (3) and (4). All these values are stored in the PC database.

And so if $i \leq p^r$, then the program continues the initial calculations. Otherwise (if $i > p^r$), the following average values are calculated: maxflow maximum flow, $f [i, j]$ flow distribution, and generalized "benefit" Φ_{zy} on each run. The following are the bottlenecks in the network: $[i, j] - c [i, j]$ and in those places where the matrix takes negative values by the amount of this difference, the capacity of the matrix $c [i, j]$ increases to provide rational organization of the railway network flows.

Conclusion

A simulation model of the transport network has been developed, taking into account one input and one output in the network.

Thus, the following particular problems were solved:

- the urgency of using the simulation model is to study transport network flows;
- compilation of lists of input and output parameters of the simulation model of the railway transport network;
- development and implementation of the simulation of model algorithm;
- solution of test problems with the help of simulation.

Thus, a simulation model of the transport network was developed and software was implemented. That implements the model of the railway network.

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Темир йўл транспорт оқимлари учун темир йўлни ташкил этишнинг имитацион моделлари

Ушбу мақолада темир йўл хабарлашувини бутун логистик занжир бўйлаб юкларни оптимал бошқаруви масалаларини ечиш, йўл харитасидаги максимал даражадан ошмайдиган ёки минимал даражадан камайиб кетмайдиган аниқ бир юкни юклаш ишларини режасини шакллантириш ва амалга ошириш жараёнларини ташкил этиш масалалари кўриб чиқилган. Бундан ташқари мақолада темир йўл тармоғида максимал оқимни топиш масаласи хар қандай транспорт тармоғининг максимал оқимининг минимал ўтказиш хусусияти тенг эканлиги асослаб берилган. Агар оқим максимал бўлса, унда тармоқнинг ўтказиш хусусияти оқимнинг кучига тенг деган тасаввур пайдо бўлиши ва бу теорема Форд-Фалкерсон алгоритми қўлланиши билан исботланиши мақолада келтирилган.

Калит сўзлар: темир йўл тармоғи, материал ва ахборот оқими, граф модели, формаллаштириш, тузилма, ўтказиш хусусияти, максимал оқим, формирование.

UDC 004.942

Sh.B.Redjepov, S.Uguz, E.Acar

TRIANGULAR VON NEUMANN CELLULAR AUTOMATA OVER GALOIS FIELD GF(2)

The fundamental structure of cellular automata (CA) is a discrete special dynamical model, but the global behaviors at many iterative times can be close nearly a continuous mathematical model and system. It is known that CA theory is a very rich and useful dynamical model by focusing on their local information and neighboring cells. The mathematical view of the basic model shows the computable values of the mathematical structure of CA. In the present paper, it is investigated the structure of two-

dimensional (2D) finite, linear, triangular von Neumann CA with null boundary over Galois field GF(2). In other words, it is considered on Galois field, i.e. 2-state case or Z_2 . Here we obtain the transition (or information) rule matrices for each special cases presented in the paper. As far as we know, there is no structure study of von Neumann 2D linear CA on triangular lattice over GF(2) in the literature. Due to main CA structures are sufficiently simple to investigate in mathematical ways, we consider that the present new construction could be applied many areas related to these CA using any other transition rules.

Keywords: Cellular automata, Galois Field, triangular lattice, transition rule matrices.

1. Introduction

Cellular automata theory (CA theory for brevity) introduced by Ulam and von Neumann [1] in the early 1950s and was systematically studied by Hedlund from mathematical perspective. One-dimensional (1D) CA has been investigated to very point of views. On the other hand a little interest was given to two-dimensional cellular automata (2D CA). Von Neumann [1] showed that a cellular automaton could have universal properties. Due to complexity of CA theory, von Neumann rules were never studied on a computer language. In the beginning of 1980s, Wolfram [2] studied in very details of a family of simple 1D CA rules and showed that even these simplest rules are capable of interesting complex behaviors. Some basic and original mathematical CA models using matrix algebra over the two states or binary field which characterize the behavior of 2D nearest neighborhood linear CA with null and periodic boundary conditions have been seen in the literature [7, 8, 9, 10, 11]. 2D CA theory has received remarkable interest and attention in the last few decades [3, 4, 5, 6, 11, 12, 13, 14, 15, 16]. Due to its striking structures, CA theory has given the opportunity to model and understand many interesting behaviors in nature easier. Here we study the theory of two dimensional uniform null boundary von

Neumann triangular CA (2D NB CA) over Galois field GF(2) (see CA structures and applications in Figs. 1-7).

In this paper, we concentrate a special family of 2D finite linear CA with null boundary condition over Galois field, i.e., the binary states field Z_2 . Here, we set up a specific relation between the structure of these CA and transition matrix rules of 2D linear CA with null boundary condition. We determine and study of the transition rule matrices of this special CA by means of the matrix algebra theory. It is known that CA nature is very simple to allow mathematical studies in dynamical systems, it is believed that these linear CA can be found many different kind of real life applications. The present results should produce further to the algebraic consequences of these 2D linear CA and relates some elegant real life applications found by the authors in the literature (see details in [11, 12, 15, 16, 17]).

The organization of the present paper is constructed as follows. In Section 2, it is given the preliminaries of CA theory and the triangular lattice structures. Section 3 presents the transition rule matrices for each cases, corresponding to the 2D von Neumann finite triangular CA. Finally conclusions are summarized in Section 4

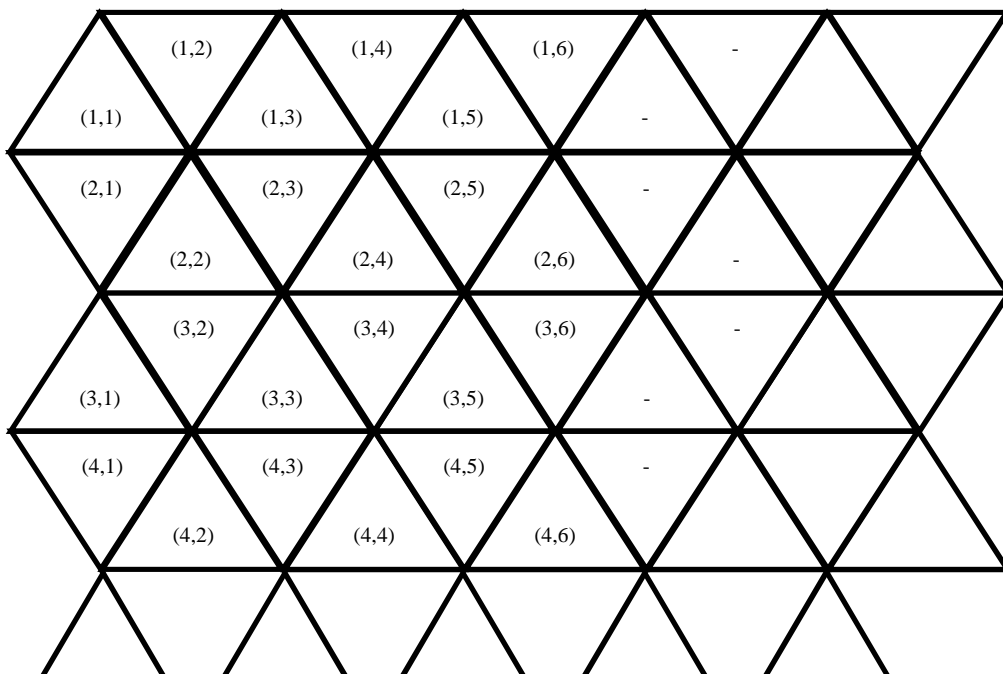


Figure 1. 2D finite CA configuration on a triangular lattice model

2.Preliminaries and triangular lattice

We study new type of lattice model, i.e. triangular lattice (see Figs. 1-2), for 2-states finite linear von

Neumann 2D CA. As far as we know, there is no construction study and methods for von Neumann 2D CA on triangular lattice over Galois field in the literature

up to now. Here, we firstly focus on special boundary condition of 2D von Neumann CA over binary field. After then we obtain the information rule matrices for special cases. Considering the neighbors of the extreme cells, there are two well-known studied neighbor approaches as below.

1. Null or 0-fixed boundary CA (NB): The borderline cells are connected to the 0-state.

2. Periodic boundary CA (PB): The borderline cells are contiguous to each other periodically in the boundaries.

In the present paper we only deal with null or fixed (0-th state) valued boundary condition. In other words, it is said that CA with null boundary case is the

borderline cells in the boundaries are considered as zero-fixed states. (i.e. The surrounding neighbor cells spin values are all 0-state, check for better understanding in [7, 15]).

It is also known that the description question of 2D CA configurations can be transformed to the description of 1D configurations by considering $m \times n$ configurations as $mn \times 1$ type configurations as follows. To obtain this procedure, it is defined the following map

$$I: M_{m \times n}(Z_2) \rightarrow Z_2^{mn} \quad (1)$$

that gets the t^{th} state $[X_t]$ given by

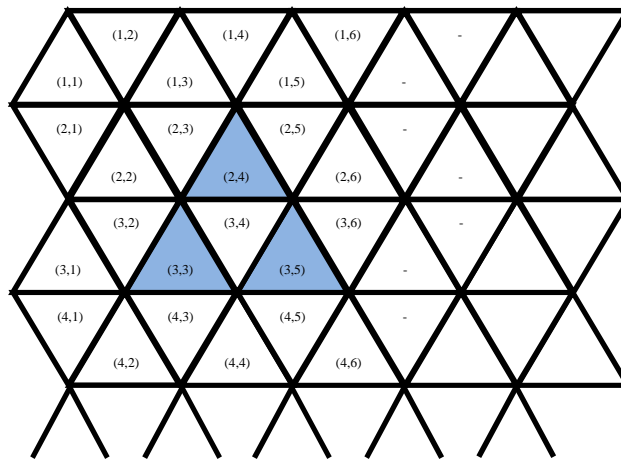


Figure 2. Von Neumann neighborhoods for the cell $x_{(3,4)}$ on the triangular lattice model

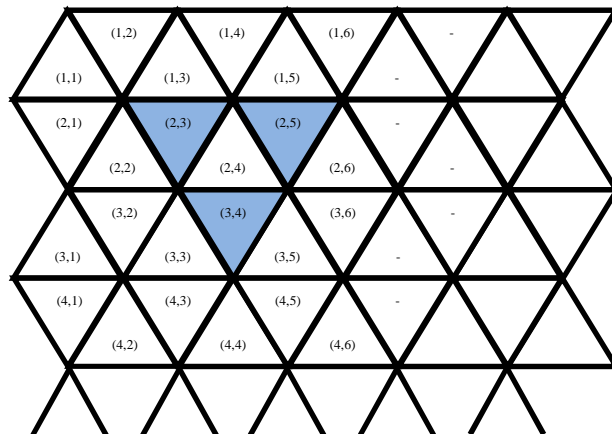


Figure 3. Von Neumann neighborhoods for the cell $x_{(2,4)}$ on the triangular lattice model

$$\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} = (x_{11}, x_{12}, \dots, x_{1n}, \dots, x_{m1}, \dots, x_{mn})^t. \quad (2)$$

Note that the superscript t indicates the transpose sign. Then the local rules are assumed to act on Z_2^{mn} on the contrary $M_{m \times n}(Z_2)$. Hence the $C^{(t)}$ matrix

$$C^{(t)} = \begin{pmatrix} x_{11}^{(t)} & \dots & x_{1n}^{(t)} \\ \vdots & \dots & \vdots \\ x_{m1}^{(t)} & \dots & x_{mn}^{(t)} \end{pmatrix} \quad (3)$$

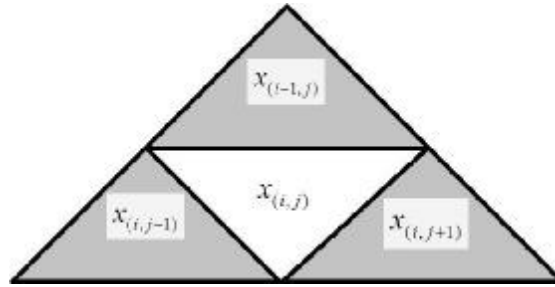


Figure 4. The nearest configurations of 2D finite von Neumann CA for the case: x_{ij} is inside of the upright triangle neighbors

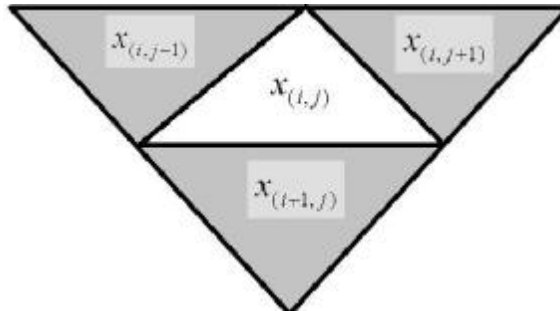


Figure 5. The nearest configurations of 2D finite von Neumann CA for the case: x_{ij} is inside of the downright triangle neighbors

is denoted the configuration matrix at time t for 2D finite CA. Using the equation (2), it can be defined as below

$$(T_{Rule})_{mn \times mn} \begin{pmatrix} x_{11}^{(t)} \\ \vdots \\ x_{1n}^{(t)} \\ \cdot \\ x_{m1}^{(t)} \\ \cdot \\ x_{mn}^{(t)} \end{pmatrix} = \begin{pmatrix} x_{11}^{(t+1)} \\ \vdots \\ x_{1n}^{(t+1)} \\ \cdot \\ x_{m1}^{(t+1)} \\ \cdot \\ x_{mn}^{(t+1)} \end{pmatrix} \quad (4)$$

For analysis and further computation, the each cell states has a spin value which takes in finite or infinite states set. Here this spin values set is chosen from Galois field GF(2).

Also it is denoted by $x_{(i,j)}^{(t)}$ as the spin states of the cell (i, j) at time $t + 1$ should be denoted by $x_{(i,j)}^{(t+1)} = y_{(i,j)}^{(t)}$. Consider the triangular transition configuration or information matrix

$$C^t = \begin{pmatrix} x_{11}^{(t)} & \dots & x_{1n}^{(t)} \\ \dots & \dots & \dots \\ x_{m1}^{(t)} & \dots & x_{mn}^{(t)} \end{pmatrix}$$

If we combine planar triangular structure with column vectors by converting them from $C^{(t)}$ to $([X]_{mn \times 1})^T = (x_{11}^{(t)}, x_{12}^{(t)}, \dots, x_{1n}^{(t)}, \dots, x_{m1}^{(t)}, \dots, x_{mn}^{(t)})^T$, then it can be considered the translation rule matrix T_{Rule} as follows.

$$(T_{Rule})_{mn \times mn} \cdot [X]_{mn \times 1} = [Y]_{mn \times 1} \quad (5)$$

$$\text{where } ([Y]_{mn \times 1})^T = (y_{11}^{(t)}, y_{12}^{(t)}, \dots, y_{1n}^{(t)}, \dots, y_{m1}^{(t)}, \dots, y_{mn}^{(t)})^T$$



Figure 6. Geometrically complex but topologically uniform cellular surface of a roof. Every triangular surface of the demonstrated buildings has three triangular neighbors

Remark 1. In the literature, the most commonly used lattice for CA is an orthogonal Z^d structure, several studies have been done to test the properties of other lattice models such as hexagonal (see [11, 12, 15, 18]). In the present work, we investigate CA studies on a new type of lattice model, i.e. triangular lattice (see Figs. 1-8), for 2-state, finite, linear, von Neumann neighborhood. An important contribution of the present study can be emphasized that there is no structure investigation for von Neumann 2D linear CA over Galois field on triangular lattice in the literature. Also note that we study von Neumann neighborhood triangular 2D CA (Figs. 1-5) and also it should be investigated Moore (see details in [18].) neighborhood 2D CA as a future direction.

Cellular automata theory can be applied in constructing of building envelope (see Fig. 6) and intelligent skin of a building (see Fig. 7) for any shape and a certain appearance. An important property that any 3D surface can be triangulated [18]. It means that it could become a grid of topologically identical elements. With the exception of boundary conditions, every triangle in the grid has exactly three neighboring triangles (see Figs. 2-3). However it can be possible to control to spin states values of the whole surface by considering the behavior of the CA theory [18].

3. The transition rule matrices for von Neumann 2D CA over triangular lattice

In the present paper, it is dealt with special finite CA for von Neumann neighbors on triangular lattice. These CA are studied under null (or fixed 0-th state)

boundary condition (NB) with the 2-state spin values, i.e. over Galois field or Z_2 . In the present section, we investigate the transition rule matrix of the triangular finite null boundary CA.

Note that there are two cases the cell $x_{i,j}$ considering for the von Neumann neighbors.

- If $x_{i,j}$ is inside of the upright triangle (see details in Fig. 4) of the von Neumann neighbors, then we get $y_{(i,j)}^{(t)} = x_{(i-1,j)}^{(t)} + x_{(i,j+1)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2}$ (6)

- If $x_{i,j}$ is inside of the downright triangle (see details in Fig. 5) of the von Neumann neighbors, then

$$y_{(i,j)}^{(t)} = x_{(i,j+1)}^{(t)} + x_{(i+1,j)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2} \quad (7)$$

where $y_{(i,j)}^{(t)} \in Z_2$.

In the next subsection, it will be obtained the transition rule matrices corresponding to the 2D finite von Neumann neighborhood CA local rule under null boundary condition. It has been studied two important special cases as presented theorems below respectively.

3.1 Rule Matrix $(T_R)_{mn \times mn}$ for m -Odd and n -Any Case

Theorem 1. Let m be odd and n any positive integers and $m, n \geq 3$. Then, the transition rule matrix from $(T_{Rule}^{m-odd})_{mn \times mn}$ from Z_2^{mn} to Z_2^{mn} which takes the t^{th} finite von Neumann CA over triangular lattice configuration $C^{(t)}$ of order $m \times n$ to the $(t+1)^{th}$ state $C^{(t+1)}$ under null boundary condition is obtained by

$$(T_{Rule}^{m-odd})_{mn \times mn} = \begin{pmatrix} K & M_1 & O & O & \dots & \dots & O & O \\ M_1 & K & M_2 & O & \dots & \dots & O & O \\ O & M_2 & K & M_1 & O & \dots & O & O \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ O & O & \dots & O & M_2 & K & M_1 & O \\ O & O & \dots & \dots & O & M_1 & K & M_2 \\ O & O & \dots & \dots & O & O & M_2 & K \end{pmatrix} \quad (8)$$

here O is the zero matrix and each sub-matrix is $n \times n$,

$$K = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 \end{pmatrix}, M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \end{pmatrix}$$

Proof 1. Denote e_{ij} as the matrix of size $m \times n$, then the (i,j) position element is equal to one and the other all entries elements equal to zero. This is well-known that the vectors e_{ij} present the standard basis elements for the matrix space. To establish the transition rule matrix $(T_{Rule}^{m-odd})_{mn \times mn}$ structure, it is needed to specify the action of $(T_{Rule}^{m-odd})_{mn \times mn}$ on the bases e_{ij} vectors elements respectively. Firstly, let us take the linear transition T from $m \times n$ matrix space structure to itself. After then, let us relate the transition T with T_M . Also consider e_{ij} , then the images of e_{ij} which is $T(e_{ij})$ are connected to the three nearest neighbors elements

considering the von Neumann neighbor (see Figs. 4-5). Hence $T(e_{ij})$ elements equals to a linear summation of its three neighbors-elements. If x_{ij} elements positions can be different within the triangular lattice, then the border elements of the location of the x_{ij} cells differ, and hence the bordering components elements of the matrix should be different (see details in Figs. 4-5). Considering the neighboring relations, these neighbors govern the rule structure, then we can observe the bordering relations as given below. First bordering relation of cell x_{ij} is inside of the upright triangle (see Fig. 4) corresponding to the von Neumann neighbors, then it is obtained

$$y_{(i,j)}^{(t)} = x_{(i-1,j)}^{(t)} + x_{(i,j+1)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2} \tag{9}$$

Second bordering relation of cell x_{ij} is inside of the downright triangle (see Fig. 5) corresponding to the von Neumann neighbors, then we have

$$y_{(i,j)}^{(t)} = x_{(i,j+1)}^{(t)} + x_{(i+1,j)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2} \tag{10}$$

where, $x_{(i,j)}^{(t)} \in Z_2$. These bordering relations of the main cell x_{ij} is summarized as follows.

$$T(e_{ij}) = e_{i-1,j} + e_{i,j+1} + e_{i+1,j+1} + e_{i+1,j} + e_{i+1,j-1} + e_{i,j-1} \pmod{2} \tag{11}$$

Thus, we obtain m,n linear equations and the transition of the representation matrix related to the linear equations is presented as in theorem.

Here we can give a specific m,n -odd case values example. We can see better the structure of the transition

rule matrix of 2D finite von Neumann CA over triangular lattice.

Example 1 Let us consider the case for $m = 3, n = 3$ (see Figs. 1-2). The rule matrix of 2D finite CA with von Neumann neighbors rule over the Galois field $GF(2)$ (i.e. the field Z_2) is found as follows,

$$(T_{Rule}^{m=3,n=3})_{9 \times 9} = \begin{pmatrix} K & M_1 & O \\ M_1 & K & M_2 \\ O & M_2 & K \end{pmatrix}$$

where K, M_1, M_2 are the sub-matrices of order 3×3 , written as

$$K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

For the structure of the transition rule matrix of 2D finite von Neumann CA over triangular lattice, we now consider m -odd and n -even case example as follows.

Example 2 Let us give the case for $m = 5, n = 4$ (see Figs. 1-3). The transition rule matrix of 2D finite von Neumann CA over triangular lattice over Galois field $GF(2)$ (or the field Z_2) is given the following way.

$$(T_{Rule}^{m=5,n=4})_{20 \times 20} = \begin{pmatrix} K & M_1 & O & O & O \\ M_1 & K & M_2 & O & O \\ O & M_2 & K & M_1 & O \\ O & O & M_1 & K & M_2 \\ O & O & O & M_2 & K \end{pmatrix}$$

where K, M_1, M_2 are the sub-matrices of order 5×4 , can be written as follows.

$$K = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.2 R rule Matrix $(T_R)_{mn \times mn}$ for m -Even and n -Any Case

Theorem 2. Let m be even and n any positive integers and $m, n \geq 3$. Then the transition rule matrix

$(T_R^{m-even})_{mn \times mn}$ from to Z_2^{mn} which takes the t^{th} finite von Neumann triangular lattice configuration $C^{(t)}$ of order $m \times n$ to the $(t+1)^{\text{th}}$ state $C^{(t+1)}$ under null boundary by

$$(T_{\text{Rule}}^{m-even})_{mn \times mn} = \begin{pmatrix} K & M_1 & O & O & \dots & \dots & O & O \\ M_1 & K & M_2 & O & \dots & \dots & O & O \\ O & M_2 & K & M_1 & O & \dots & O & O \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ O & O & \dots & O & M_2 & K & M_2 & O \\ O & O & \dots & \dots & O & M_1 & K & M_1 \\ O & O & \dots & \dots & O & O & M_1 & K \end{pmatrix} \quad (12)$$

where O is the zero matrix and all sub-matrix is $n \times n$.

Proof 2 To obtain the transition rule matrix $(T_{\text{Rule}}^{m-even})_{mn \times mn}$ structure, it is needed to specify the action of $(T_{\text{Rule}}^{m-even})_{mn \times mn}$ on the bases e_{ij} vectors elements respectively. Consider e_{ij} , the images of e_{ij} which is $T(e_{ij})$ are connected to the three nearest neighbors elements considering the von Neumann

neighbor (see Figs. 4-5). Then $T(e_{ij})$ elements equals to a linear summation of its three neighbors-elements. Considering the neighboring relations, these neighbors govern the rule structure, then we can observe the bordering relations same in Theorem 1. First bordering relation of cell x_{ij} is inside of the upright triangle (see Fig. 4), then

$$y_{(i,j)}^{(t)} = x_{(i-1,j)}^{(t)} + x_{(i,j+1)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2} \quad (13)$$

Second bordering relation of cell x_{ij} is inside of the downright triangle (see Fig. 5), then

$$y_{(i,j)}^{(t)} = x_{(i,j+1)}^{(t)} + x_{(i+1,j)}^{(t)} + x_{(i,j-1)}^{(t)} \pmod{2} \quad (14)$$

where $x_{(i,j)}^{(t)} \in Z_2$. Finally we obtain the transition of the representation matrix presented as in Theorem 2.

Consider m -even and n -odd case example for the structure of the transition rule matrix of 2D finite von Neumann CA over triangular lattice as below.

Example 3 Let us take $m = 4$, $n = 5$ (see Figs. 1-3). The transition rule matrix of 2D finite CA with von Neumann triangular neighbors over Galois field $GF(2)$ i.e. Z_2 , is given as follows:

$$(T_{\text{Rule}}^{m=4,n=5})_{20 \times 20} = \begin{pmatrix} K & M_1 & O & O \\ M_1 & K & M_2 & O \\ O & M_2 & K & M_1 \\ O & O & M_1 & K \end{pmatrix}$$

where K, M_1, M_2 are the sub-matrices of order 5×5 , where

$$K = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



Figure 7. The skin of a building in architecture is an application area of triangular CA

Remark 2 It can be investigated the CA works on a triangular lattice (or triangular grid works) on both regular and irregular lattices as in [18]. The only difference is the coordinates of the vertices, while the topological information about the graphic data remains the same. (See details in [18]). Each triangle in the grid works have exactly 3-neighboring triangles (see Figs. 2-3). Regarding CA structure, each element of a triangular surface could be assigned with special characteristic values, such as, color pixel values or gray scale levels for similarity to CA states or spin values. Hence it can be possible to control the spin states of the constructed surface behavior.

4 Conclusion

In the present paper we investigate the theory two dimensional, uniform null boundary, von Neumann CA over Galois field $GF(2)$. As far as we know, there is no structure study of von Neumann 2D linear CA on triangular lattice over $GF(2)$ in the literature. Due to main CA structures are sufficiently simple to investigate in mathematical ways, the present new construction could be applied many areas related to these CA using any other transition rules. It is introduced two main theorems for determining the structure of these triangular von Neumann CA for a general case of linear transformation. Also after constructed the transition rule matrix representation of 2D linear von Neumann CA, one should find some real life applications for the 2D linear CA, it is another goals of the next study. We believe that triangular CA theory could be applied successfully in especially image processing area [12, 13, 14, 15, 16] and the other science branches in near future [7, 8, 10]. Some other interesting results and further connections on this direction wait to be explored in triangular von Neumann 2D CA [18].

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Ш.Б.РЕДЖЕПОВ, С.УГУЗ, Э.АЧАР

Треугольные клеточные автоматы Фон Неймана над полями ГАЛУА GF(2)

Фундаментальная структура клеточных автоматов (КА) представляет собой дискретную специальную динамическую модель, но глобальное поведение во многих итерационных временах может быть близким к непрерывной математической модели и системе. Известно, что теория КА - очень богатая и полезная динамическая модель, фокусируясь на их локальной информации и соседних ячейках. Математическое представление базовой модели показывает вычислимые значения математической структуры КА. В настоящей работе исследована структура двумерного (2D) конечного линейного треугольного клеточного автомата фон Неймана с нулевой границей над полем Галуа GF(2).

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Э.С.Бабаджанов

АХБОРОТ ТИЗИМЛАРИДА САМАРАЛИ ЭЛЕКТРОН ХИЗМАТЛАРНИ ТАНЛАШ ТЕХНОЛОГИЯСИ

Мақолада хизматлар мажмуасидан иборат ахборот тизимларда фойдаланувчининг ҳуқуқ-даражалари бўйича реал вақтда муҳим электрон хизматларни самарали танлаш масаласи кўриб чиқилади. Хизмат кўрсатишда мавжуд объектлараро муносабатларида пайдо бўладиган аломатларнинг жараён схемаси ишлаб чиқилган. Жараёндаги ҳар бир объектга эксперт коэффициентларига эга параметрли белгилашлар киритилади. Хизматларни самарали танлашда хизматларни муддати, жорий вақтга нисбати, боғлиқлиги, бажариш хажмига ва салмоқ коэффициентларини мезонли ўзгартириш орқали хизмат муҳимлигини ошириш технологияси ишлаб чиқилади. Хизматлар салмоғини ўзгартирувчи технология учун махсус мезонлар, функциялар ва алгоритмлар ишлаб чиқилган. Хизматларни тақдим этувчи ахборот тизимларда фойдаланувчига самарали хизматларни танлаш технологиясидан фойдаланиш тақлиф этилади.

Калит сўзлар: интерактив ахборот тизими, электрон хизмат, жараён, функция, алгоритм, синфлаштириш, объект, аломат.

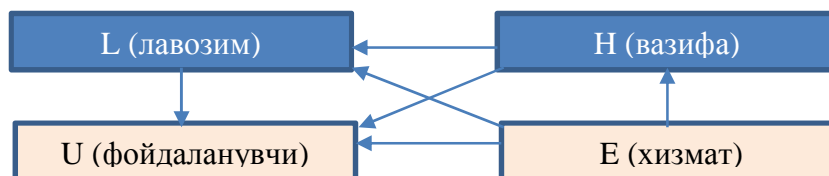
КИРИШ

Тадқиқ қилинадиган соҳа ташкилотлари фаолиятини том маънода мақсадга йўналтирилган хизматлар мажмуаси сифатида қараш мумкин. Ташкилот доирасидаги ахборот тизимларни реал ишчи электрон хизматлар (ЭХ) билан таъминлаш учун зарур бўлган маълумотларнинг объектлари танланади, кейин ушбу маълумотларни қайта

Другими словами, он рассматривается на поле Галуа, то есть в случае с 2 состояниями или Z_2 . Здесь мы получаем матрицы правил перехода (или информации) для каждого отдельного случая, представленного в статье. Насколько нам известно, в литературе отсутствует структурное исследование двумерного линейного КА фон Неймана на треугольной решетке над GF(2). Из-за того, что основные структуры КА достаточно просты для изучения математически, мы считаем, что в настоящее время новое строительство может быть применено во многих областях, связанных с этими КА, с использованием любых других правил перехода.

Ключевые слова: клеточные автоматы, поле Галуа, треугольная решетка, матрицы правил перехода.

ишловчи хизматлар субъектларга тақдим этилади. Ахборот тизим (АТ) ташкилот фаолиятини автоматик бажариш эмас, балки маълумотлар оқимини ва у орқали электрон хизматлар мажмуасини бошқариш вазифасини бажаради. Тизимдаги барча хизматлари фойдаланувчиларга йўналтирилган бўлиб, у лавозимлар ва лавозимлардаги вазифалар орқали кўрсатилади (схемада келтирилган).



Хизматлар вазифаларга, вазифалар лавозимларга, лавозимлар фойдаланувчиларга ва шунингдек, хизматлар лавозим ва фойдаланувчиларга, вазифалар фойдаланувчиларга бириктирилади. Бундан хизматларни тақдим этиш қуйидаги синфдаги шакллар орқали амалга оширилади: 1) хизмат тўғридан-тўғри фойдаланувчи, вазифа ва лавозимга; 2) хизмат тўғридан-тўғри вазифа орқали фойдаланув-

чи ва лавозимга; 3) хизмат фойдаланувчига лавозим орқали кўрсатилади.

МАСАЛАНИНГ ҚЎЙИЛИШИ

Реал вақтда фойдаланувчига АТда тақдим этилаётган муҳим электрон хизматларни тизимли саралаб, автоматик тақлиф этиш хизматларни интеллектуал самарали танлаш масаласини келтириб чиқаради.