UDK 539.3

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Mathematical and numerical models of thermoelastic plates of complex configuration

Annotation. Mathematical and numerical models of thermoelastic plates of complex configuration are discussed in the paper. The basic equations are determined of two-dimensional thermoelasticity in a quasistatic statement, stationary thermal conductivity of a plate; a mathematical model of thermoelastic plates is determined. A computational algorithm for calculating magnetoelastic plates of complex configuration is developed using a combination of V.L. Rvachev R-function method and Bubnov-Galerkin method. A computational algorithm for calculating thermoelastic plates of complex configuration is described. Computational experiments were carried out to calculate thermoelastic plates of complex configuration. The results of computational experiments are given in the form of tables.

Keywords: Mathematical models, numerical models, thermoelastic plates, complex configuration, Rvachev R-function method, Bubnov-Galerkin method.

Introduction. The solution of problems of modern information technologies includes a large amount of research on algorithms and programs, and an analysis and processing of results. The method of scientific research and engineering calculations, which provides a solution to important class of problems is called a computational experiment. Currently, the methods of mathematical simulation are used to study various problems in economy posed by the engineering practice in mathematical physics and related to engineering calculations of various physical and mechanical fields, such as temperature, force, strain fields, which determine the main qualitative characteristics of products (transfers reliability, durability, etc.) [1-7]. So, this article is devoted to the mathematical simulation of thermoelastic plates of complex configuration.

The problem of thermal stress state arises in the most diverse fields of engineering and is one of the main problems in strength of structure elements operating in conditions of non-uniform and non-stationary heating. Under non-uniform and non-stationary heating, the physicomechanical properties of materials change, the temperature gradients arise that cause thermal stresses. Knowledge of the magnitude and nature of thermal stresses is necessary for a comprehensive analysis of the structure strength.

In recent years, practical importance of study of structure element vibrations caused by a variable heat load has grown in connection with the use of flexible power elements on satellites, and the needs of nuclear power ingineering.

Variation principles that allow us to obtain differential equations and approximate solutions to specific problems are of great importance in the theory of thermoelasticity. The first variation principle of linear coupled thermoelasticity was developed by M. Biot. By analogy with the results of the isothermal theory of elasticity, this investigation was generalized by research in thermoelasticity.

The quasistatic problems of thermoelasticity used in practice are the plane problems of thermoelasticity, the thermoelasticity of circular plates and shells of revolution, and the axisymmetric thermoelasticity problems.

In practice, the problems of complex calculation of physicomechanical fields are numerous, they include the processes of thermoelastic thin plates oscillation, where the interaction of temperature and strain fields is considered.

Due to the complex calculation of physical and mathematical fields by mathematical models, such processes enter the group of interconnected boundary value problems.

In this case, we are talking about an uncoupled problem of thermoelasticity of thin plates. First the problem of fluctuation of unsteady heat conductivity of plates is solved and the temperature T is determined. Then, the value of T, as the mass force enters the right side of the equation of plate motion, which in turn is solved under initial and corresponding boundary conditions depending on how the plate edges are fixed.

The statement of the problem of thermoelastic plates of complex configuration. First, consider mathematical models of heat conduction processes.

We set the middle surface of the thin plate in $x0y$ plane of the Cartesian coordinate system. Denote the plate thickness by h. Consider a plate under non-stationary convective heat transfer on its contour Γ and on surfaces $z=\pm\frac{h}{2}$ (Fig. 1).

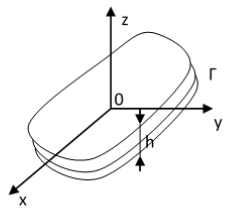

Fig. 1. Plate under non-stationary convective heat transfer

Mathematical model of thermoelastic plates of complex configuration. Determination of the non-stationary temperature field of such a plate at constant thermophysical characteristics is reduced to solving an equation of the form [8]:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{a}\frac{\partial T}{\partial t} \tag{1}$$

The solution of equation (1) must satisfy the following initial and boundary conditions

$$T^{(0)}\Big|_{t=0}=T_0 \tag{2}$$

$$\left.\begin{aligned}&\left[\frac{\partial T}{\partial n}+\frac{\alpha_\Gamma}{\lambda_q}\left(T-\vartheta_L\right)\right]_\Gamma=0;\\&\left[\frac{\partial T}{\partial z}+\frac{\alpha_3}{\lambda_q}\left(T-\vartheta_3\right)\right]_{z=\frac{h}{2}}=0;\\&\left[\frac{\partial T}{\partial z}+\frac{\alpha_4}{\lambda_q}\left(T-\vartheta_4\right)\right]_{z=-\frac{h}{2}}=0\end{aligned}\right\} \tag{3}$$

In equations (1) - (3) the following notations are introduced:

T is the plate temperature; T_0 is the initial temperature of the plate; $\vartheta_\Gamma, \vartheta_3,$ *and* ϑ_4 are the temperatures of the medium on the plate contour Γ, on the surfaces $z = \frac{h}{2};$ *and* $z = -\frac{h}{2};$ $\vartheta_\Gamma, \vartheta_3,$ *and* ϑ_4, respectively, α_L, α_3 *and* α_4 are the heat transfer coefficients on the plate contour Γ, on the surfaces $z = \frac{h}{2};$ *and* $z = -\frac{h}{2};$ and α is the coefficients of heat and temperature conductivity of the plate material, *n* is the external normal to the plate.

When drawing up the third boundary condition (4), we take into account that the direction of the external normal to the surface $z = -\frac{h}{2}$ is opposite to the positive direction of the z axis.

Reducing the three-dimensional non-stationary heat conductivity problem (1) to the two-dimensional one an approximate solution to this problem is determined. Approximating the temperature distribution T over the plate thickness by a power law

$$T = \sum_{j=0}^{m} T^{(j)}(x, y, t) z^j \tag{4}$$

the problem under consideration is reduced to the two-dimensional one.

To derive an equation satisfying the functions $T^{(j)}$, we multiply equation (1) by, $z^p\,(p = 0, 1, ..., m)$, and integrate it over z from $-\frac{h}{2}$; to $\frac{h}{2}$, taking into account the identity

$$z^p \frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left(z^p \frac{\partial T}{\partial z} - p z^{p-1} T \right) + p(p-1) z^{p-2} T \tag{5}$$

and boundary conditions (3). As a result we get:

$$\nabla^2 \Theta_p - z^p \frac{h^{p-1}}{2^p} \left\{ 2p\left[T\left(\frac{h}{2}\right) + (-1)^p T\left(-\frac{h}{2}\right) \right] + \gamma_3 \left[T\left(\frac{h}{2}\right) - \vartheta_3 \right] + \right. \tag{6}$$

$$\left. (-1)^p \gamma_4 \left[T\left(-\frac{h}{2}\right) - \vartheta_4 \right] \right\} + p(p-1)\Theta_{p-2} = \frac{1}{a}\frac{\partial \Theta_p}{\partial t},$$

$$(p = \overline{0, m})$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2};\ \Theta_p = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^p T dz;\ \gamma_3 = \frac{\alpha_3 h}{\lambda_q};\ \gamma_4 = \frac{\alpha_4 h}{\lambda_q}$$

Performing similar operations with condition (2) and the first one of conditions (3), we find the following boundary conditions for function Θ_p:

$$\left[\frac{\partial \Theta_p}{\partial n} + \frac{\alpha_\Gamma}{\lambda_q}\left(\Theta_p - \theta_p \right) \right]_\Gamma = 0 \tag{7}$$

where

$$\Theta_{0p} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^p T_0 dz,\ \theta_p = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^p \vartheta_\Gamma dz$$

Varying the values of *p* in equation (4) and conditions (7), we can compose a system of equations describing the non-stationary thermal conductivity of the plate for different power laws of temperature change along the plate thickness. We present these equations and the corresponding boundary conditions in the case when the quantities T_0 *and* ϑ_Γ are independent of z.

Assuming temperature to be constant along the plate thickness

$$T = T^{(0)} \tag{8}$$

and assuming that $p = 0$ we get the heat conduction equation:

$$\nabla^2 T^{(0)} - \frac{2\gamma}{h^2}\left(T^{(0)} - \vartheta \right) = \frac{1}{a}\frac{\partial T^{(0)}}{\partial t}, \tag{9}$$

where

$$\gamma = \frac{(\alpha_3 + \alpha_4)h}{2\lambda_q},\ \vartheta = \frac{\alpha_3 \vartheta_3 + \alpha_4 \vartheta_4}{\alpha_3 + \alpha_4}$$

Under conditions

$$T^{(0)}\Big|_{t=0} = T_0 \tag{10}$$

$$\left[\frac{\partial T^{(0)}}{\partial n} + \frac{\alpha_\Gamma}{\lambda_q}\left(T^{(0)} - \vartheta_\Gamma \right) \right]_\Gamma = 0 \tag{11}$$

Assuming a linear law of temperature variation along the plate thickness

$$T = T^{(0)} + T^{(1)} z \tag{12}$$

corresponding to the values $p = 0$ *and* $p = 1$, we find a system of two equations of thermal conductivity:

$$\nabla^2 T^{(0)} - \frac{2\gamma}{h^2}\left(T^{(0)} - \vartheta \right) - \frac{\gamma_3 - \gamma_4}{2h} T^{(1)} = \frac{1}{a}\frac{\partial T^{(0)}}{\partial t}; \tag{13}$$

$$\nabla^2 T^{(1)} - \frac{6(\gamma_3 - \gamma_4)}{h^3} T^{(0)} - \frac{6(2+\gamma)}{h^3}\left(T^{(1)} - \mu \right) = \frac{1}{a}\frac{\partial T^{(1)}}{\partial t}, \tag{14}$$

where

$$\mu = \frac{\gamma_3 \vartheta_3 + \gamma_4 \vartheta_4}{(2+\gamma)h}$$

Under conditions

$$T^{(0)}\Big|_{t=0} = T_0, \qquad T^{(1)}\Big|_{t=0} = T_0 \tag{15}$$

$$\left[\frac{\partial T^{(0)}}{\partial n} + \frac{\alpha_\Gamma}{\lambda_q}\left(T^{(0)} - \vartheta_\Gamma \right) \right]_\Gamma = 0,\ \left[\frac{\partial T^{(1)}}{\partial n} + \frac{\alpha_\Gamma}{\lambda_q} T^{(1)} \right]_\Gamma = 0 \tag{16}$$

Assuming a quadratic law of temperature variation along the plate thickness

$$T = T^{(0)} + T^{(1)} z + T^{(2)} z^2, \tag{17}$$

corresponding to the values of $p = 0, p = 1$ *and* $p = 2$, after some transformations we get the system of three equations of thermal conductivity [8]:

$$\nabla^2 T^{(0)} + \frac{3\gamma}{h^2}\left(T^{(0)} - \vartheta\right) + \frac{3(\gamma_3 - \gamma_4)}{4h} T^{(1)} + \frac{3\gamma + 20}{4} T^{(2)} = \frac{1}{a}\frac{\partial T^{(0)}}{\partial t}; \quad (18)$$

$$\nabla^2 T^{(1)} - \frac{6(2+\gamma)}{h^2}\left(T^{(1)} - \mu\right) - \frac{6(\gamma_3 - \gamma_4)}{h^3}\left(T^{(0)} + \frac{h^2 T^{(2)}}{4}\right) = \frac{1}{a}\frac{\partial T^{(1)}}{\partial t}; \quad (19)$$

$$\nabla^2 T^{(2)} + \frac{60\gamma}{h^2}\left(T^{(0)} - \vartheta\right) - \frac{15(\gamma_3 - \gamma_4)}{h^3} T^{(1)} - \frac{15(4+\gamma)}{h^2} T^{(2)} = \frac{1}{a}\frac{\partial T^{(2)}}{\partial t} \quad (20)$$

At initial conditions

$$T^{(0)}\Big|_{t=0} = T_0^{(0)}, \quad T^{(1)}\Big|_{t=0} = T^{(2)}\Big|_{t=0} = 0 \quad (21)$$

and boundary conditions

$$\left[\frac{\partial T^{(0)}}{\partial n} + \frac{\alpha_\Gamma}{\lambda_q}\left(T^{(0)} - \vartheta_\Gamma\right)\right]_\Gamma = 0,$$
$$\left[\frac{\partial T^{(1)}}{\partial n} + \frac{\alpha_\Gamma T^{(1)}}{\lambda_q}\right]_\Gamma = 0, \quad (22)$$
$$\left[\frac{\partial T^{(2)}}{\partial n} + \frac{\alpha_\Gamma T^{(2)}}{\lambda_q}\right]_\Gamma = 0$$

At identical heat transfer coefficients $\alpha_3 = \alpha_4 = \alpha$, and, consequently, identical parameters $\gamma_3 = \gamma_4 = \gamma$, equations (17), (18), (19) and (20) are, respectively:

$$\nabla^2 T^{(0)} + \frac{2\gamma}{h^2}\left(T^{(0)} - \vartheta\right) = \frac{1}{a}\frac{\partial T^{(0)}}{\partial t}; \quad (23)$$

$$\nabla^2 T^{(1)} - \frac{6(2+\gamma)}{h^2}\left(T^{(1)} - \mu\right) = \frac{1}{a}\frac{\partial T^{(1)}}{\partial t} \quad (24)$$

$$\nabla^2 T^{(0)} + \frac{3\gamma}{h^2}\left(T^{(0)} - \vartheta\right) + \frac{3\gamma + 20}{4} T^{(2)} = \frac{1}{a}\frac{\partial T^{(0)}}{\partial t}; \quad (25)$$

$$\nabla^2 T^{(1)} - \frac{6(2+\gamma)}{h^2}\left(T^{(1)} - \mu\right) = \frac{1}{a}\frac{\partial T^{(1)}}{\partial t}; \quad (26)$$

$$\nabla^2 T^{(2)} - \frac{60\gamma}{h^4}\left(T^{(0)} - \vartheta\right) - \frac{15(4+\gamma)}{h^2} T^{(2)} = \frac{1}{a}\frac{\partial T^{(2)}}{\partial t} \quad (27)$$

This problem is reduced to integrating equation [8]:

$$m\frac{\partial^2 W}{\partial t^2} + D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) = -\alpha_T\left(1 + \upsilon_0\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \quad (28)$$

$$W(x, y, t)\Big|_{t=t_0} = W_0(x, y), \ \dot{W}(x, y, t)\Big|_{t=t_0} = \dot{W}_0(x, y) \quad (29)$$

and boundary conditions depending on how the plate ends are fixed:

$$L_i W\Big|_{\Gamma_i} = \psi_i, \quad i = \overline{1, \ n}, \quad (30)$$

where α_T is the coefficient of linear dilatation (expansion) of materials;

υ_0 - Poisson's ratio; L_i - given differential operator; Γ_i - coverage of the domain boundary $\Gamma\left(i = \overline{1, \ n}\right)$ i.e. $\Gamma = \bigcap_{i=1}^{n} \Gamma_i$; ψ_i are the functions defined in the domains Γ_i.

Here, the temperature field T is determined by the solution of a boundary value problem of the form

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \alpha T = f(x, y), \quad (31)$$

$$\left(\frac{\partial T}{\partial \upsilon} + kT\right)\Big|_\Gamma = g(x, y), \quad (32)$$

where $k = \frac{\alpha}{\lambda}$, α is the heat transfer coefficient, λ is the coefficient of thermal conductivity of the material.

Boundary condition (32) characterizes the law of heat transfer between the surface of the body and the surrounding medium under heating.

As seen, the solution of the problems of thermoelastic plates is divided into two stages: the temperature field T is determined and then the deflection W, bending moments M_x, M_y and torque M_{xy} under the influence of temperature.

Computational algorithm for thermoelastic plates of complex configuration. A computational algorithm for thermoelastic plates of complex configuration is realized using a combination of the Bubnov – Galerkin methods [9–11] and V.L. Rvachev algebraic-logical method of ***R***-function [12].

For this purpose, consider particular cases of boundary conditions given in the first paragraph [8]:

a) the first principal task (the Dirichlet problem), at the boundary of the body Γ the temperature distribution is set

$$u\Big|_\Gamma = \phi_0. \quad (33)$$

A boundary value problem of this kind is considered when, due to intense heat transfer, the temperature on the surface of the body of external medium is different;

b) the second principal task (the Neumann problem), the value of the heat flow for each surface of the body is set

$$\frac{\partial u}{\partial n}\Big|_\Gamma = \phi_0 \quad (34)$$

where n is the normal to Γ, ϕ_0 – is the flow density on the surface of the body.

Such boundary-value problems usually result in problems of body heating Ω by external sources of a given intensity ϕ_0. At $\phi_0 = 0$, the condition is called the condition of thermal insulation;

c) the third principal task (the problem with an oblique derivative), a condition of this kind characterizes the heat transfer between the surface of the body and the medium under heating. In this case, the ambient temperature T_c is set and the law of heat transfer between the surface of the body and the medium is valid:

$$\frac{\partial u}{\partial n}\Big|_\Gamma = \frac{\alpha_0}{k}(T_c - T_0)$$

where α_0 – is the coefficient of heat transfer.

This condition is re-written as:

$$\left(\frac{\partial T}{\partial u} + hT\right)\Big|_\Gamma = q_1, \quad (35)$$

where

$$h = \frac{\alpha_0}{k}, \ q_1 = \frac{\alpha_0}{k} T_c;$$

d) conditions of the fourth kind express the thermal interaction between the contacting bodies. The simplest version of these conditions expresses the equality of temperatures and heat flows on both sides of the material interface. In this case, the medium conjugation condition is written as follows

$$k_1\frac{\partial T_1}{\partial n}=k_2\frac{\partial T_2}{\partial n},\ T_1=T_2, \quad (36)$$

where T_1, T_2, k_1, k_2 are the temperatures and thermal conductivity of contacting media, respectively. It follows from (36) that the solution of the conjugate problem is related to finding temperature fields on both sides of the material interface.

The boundary conditions can be of a mixed type, when a condition of one kind is specified on a part of the body surface, and of a different kind - on the remaining part. There may be various combinations of the above principal types of boundary conditions. Note the most common in practice: temperature is set on a part of the body surface, and the remaining part of this surface is thermally insulated, i.e.

$$\frac{\partial T}{\partial n}=0 \quad on \quad \Gamma_2,\ T=T_0 \quad on \quad \Gamma_1 \quad (37)$$
$$(\Gamma=\Gamma_1\cup\Gamma_2).$$

Solution structures for the above boundary conditions (33) - (37) have the form [12]:

$$T=\omega\Phi; \quad (38)$$

$$T=\Phi_0-\omega D_1\Phi_0+\omega\phi_0; \quad (39)$$

$$T=\Phi_0+\frac{\omega}{a_0}-\omega D_1\Phi_0-\omega h_0\Phi_0-\omega q_1; \quad (40)$$

$$\begin{cases} T_1=\Phi_0-\dfrac{k_1-k_2}{k_1}\omega_{12}D_1\Phi_0+\omega_{12}^2\psi_1, \\ T_2=\Phi_0. \end{cases} \quad (41)$$

$$T=T_0+\omega_1\Phi_0-\omega D_1^{(2)}(T_0+\omega_1\Phi_0). \quad (42)$$

Here Φ_0 is an arbitrary function, index $_2$ indicates that the operator D_1 is taken by function ω_2.

The structure of problem (31) - (32) solution has the form [12]:

$$T\equiv\Phi-\omega\left(D_1\Phi+k\Phi\right)+\omega g;$$

where Φ is an indeterminate function, presented in the form:

$$\Phi=\sum_{i=0}^{n}\sum_{j=0}^{i}C_{ij}\,X_i(x)\,Y_j(y),$$

where C_{ij} are the unknown coefficients to be determined; $X_i(x), Y_j(y)$ is the complete system of basic polynomials (power, trigonometric, Chebyshev, etc.).

$$D_1=\frac{\partial\Phi}{\partial x}\frac{\partial\omega}{\partial x}+\frac{\partial\Phi}{\partial y}\frac{\partial\omega}{\partial y}$$

Further, applying the Bubnov-Galerkin method, we obtain a system of algebraic equations of the form

$$AC=F \quad (43)$$

here

$$A=\{a_{ij}\}=\left\{-\iint_\Omega\left(\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right)-\alpha T_i\right)T_j\,d\Omega\right\},$$

$$F=\{f_i\}=\left\{\iint_\Omega f\,T_j\,d\Omega\right\}$$

System (43) can be solved by the Gauss method or least squares or inversion method. Computational experiments are carried out with respect to the number of coordinate functions (CF), the number of Gaussian nodes (GN). As a result, a stable value of the temperature field T is selected.

Then the problem (29) is solved.

The solution to this problem is:

$$W(x,y,t)=\sum_{i=0}^{n}\sum_{j=0}^{i}C_{ij}(t)W_{ij}(x,y) \quad (44)$$

Substituting (44) into (28) and applying the procedure of the Bubnov-Galerkin method, a system of ordinary differential equations is obtained:

$$A\ddot{C}+BC=F \quad (45)$$

At initial conditions

$$C\big|_{t=t_0}=C_0,\quad \dot{C}\big|_{t=t_0}=\dot{C}_0, \quad (46)$$

where

$$A=\{a_{ij}\}=\left\{\iint_\Omega W_i\,W_j\,d\Omega\right\},$$

$$B=\{b_{ij}\}=\left\{\iint_\Omega \nabla W_i\,W_j\,d\Omega\right\},$$

$$q=-\alpha_T(1+\upsilon_0)\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right),$$

$$C_0=A^{-1}\left\{\iint_\Omega W_0\,W_j\,d\Omega\right\},\ \dot{C}_0=A^{-1}\left\{\iint_\Omega \dot{W}_0\,W_j\,d\Omega\right\}$$

The resulting system of ordinary differential equations (45) at initial conditions (46) is solved by the Newmark method or the method of square sums.

The values of $C_{ij}(t)$ are substituted in (44) and the deflection W, bending moments, M_x, M_y and torque M_{xy} are determined. Here, the software package is supplemented with appropriate software modules for solving this problem [13-18].

The convergence of the computational algorithm was investigated with a change in the type of polynomials (power, trigonometric, Chebyshev), the number of coordinate functions and the number of Gaussian nodes.

A numerical model of thermoelastic plates of complex configuration. Consider the problem of bending of a single circular rigidly fixed thermoelastic plate. Let the temperature field be determined by the following equation at the boundary condition:

$$\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}-4T=4\left(1-x^2-y^2\right), \quad (47)$$

$$\left(\frac{\partial T}{\partial\upsilon}+3T\right)\bigg|_\Gamma=1 \quad (48)$$

Stress state of the plate is determined from the solution of the following problem:

$$D\nabla^4 W = -\alpha_T \left(1+\upsilon_0\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \quad (49)$$

$$\left.\begin{array}{l} W\big|_{\Gamma}=0, \dfrac{\partial W}{\partial \upsilon}=0 \end{array}\right\} \quad (50)$$

It is not difficult to check that the exact solution to problem (47) - (48) has the form:

$$T(x,y)=x^2+y^2$$

As seen from the table, the approximate solution obtained completely agrees with the exact solution, which proves the truth of the way we took for solving the problem of thermoelastic plates.

Table 1.

Results of the problem of bending of a single circular rigidly fixed thermoelastic plate

x, y	(CF, GC)	Q	W^*	M_x	M_y
(0,0;0,0)	Exact sol-n	40	00625	03250	03250
	(10, 10)	40	00625	03250	03250
	(15, 20)	40	00625	03250	03250
(0,0;0,0)	Exact sol-n	40	00529	02730	02730
	(10, 10)	40	00529	02730	02730
	(15, 20)	40	00529	02730	02730

The following notations are given in the tables:

$$Q=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}, \quad W^*=\frac{D}{\alpha_T\left(1+\upsilon_0\right)}W$$

Further, problems (47) - (48), (49) - (50) are solved for other types of plates, for example, for a square plate with a circular hole. Here, the convergence is investigated with respect to the number of coordinate functions (CF) and the number of Gaussian nodes (GC). The results of the computational algorithm are shown in Tables 1, 2.

Table 2.

Results of deflection W, bending moments M_x, M_y and torque M_{xy} of a single circular rigidly fixed thermoelastic plate

x, y	(CF, GC)	Q	W	M_x	M_y	M_{xy}
(0.2;0.2)	(10,10)	8458,83	0,0003427	0,0594	0,0589	-0,0126
	(15,20)	8457,12	0,0003421	0,0583	0,0575	-0,0119
(0.5;0.5)	(10,10)	167,005	0,001703	0,0357	0,0127	-0,0015
	(15,20)	166,871	0,001701	0,0348	0,0119	-0,0009
(0.8;0.8)	(10,10)	5674,70	0,000439	-0,0036	0,0143	-0,0147
	(15,20)	5669,85	0,000432	-0,0031	0,0138	-0,0135

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Conclusions. The basic equations are determined: of two-dimensional thermoelasticity in a quasistatic statement; of stationary thermal conductivity of a plate. A mathematical model of thermoelastic plates is determined. A computational algorithm for calculating magnetoelastic plates of complex configuration is developed using a combination of the Bubnov-Galerkin method and Rvachev R-function method. Computational experiments were carried out to calculate thermoelastic plates of complex configuration.

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Математические и численные модели термоупругих пластин сложной конфигурации

Аннотация. В статье обсуждаются математические и численные модели термоупругих пластин сложной конфигурации. Определены основные уравнения двумерной термоупругости в квазистатическом утверждении, стационарной теплопроводности пластины; определена математическая модель термоупругих пластин. Вычислительный алгоритм для расчета магнитоупругих пластин сложной конфигурации разработан с использованием комбинации В.Л. Метод R-функции Рвачева и метод Бубнова-Галеркина. Описан вычислительный алгоритм расчета термоупругих пластин сложной конфигурации. Проведены вычислительные эксперименты для расчета термоупругих пластин сложной конфигурации. Результаты вычислительных экспериментов приведены в виде таблиц.

Ключевые слова: математические модели, численные модели, термоупругие пластины, комплексная конфигурация, метод R-функции Рвачева, метод Бубнова-Галеркина.