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MODELING OF TRANSITION PROCESSES DURING PIPELINE TRANSPORTATION OF REAL GASES

Khujaev I.K., Mamadaliyev H.A., Aminov H.H.
husniddin_m1@bk.ru
Scientific and Innovation Center of Information and Communication Technologies,
17A Buz-2, Tashkent, 100125, Uzbekistan

The problems of identifying gas leak points, high pressure points and other deviations of the pipeline network are directly related to transition processes. Researches in transition processes in pipeline transportation are relevant both from a theoretical and from a practical point of view.

Mathematical modeling of transition processes in main pipelines is performed within the framework of quasi-one-dimensional equations of Zhukovsky N.E. taking into account the forces of friction, gravity and inertia. For the gaseous medium, these equations are in the third order with respect to unknowns. By using the method of type averaging, developed by Charny I.A. the degree of equations can be reduced by one order. And the transition to mass flow, analogous to the current function when solving two-dimensional hydrodynamic equations, allows us to obtain linear equations for mass flow, in terms of the product of unknowns of gas density and velocity. Regarding mass flow and pressure, autonomous equations are compiled that represent the type of telegraph equation.

Taking into account possible abrupt changes of the unknowns in time and distance, the solution is sought in the form of functional series. The demonstrativeness of this method lies in the fact that the perturbation frequencies relevant to the parabolic and hyperbolic types of equations and the intermediate variant are distinguished.

A general method is presented for solving problems of transition from one steady state of operation of a site to another steady state of operation. Under the steady state we refer to the stationary and periodic modes of operation of the site. In these cases, derivatives are easily determined, and the integrals involved in the general solution of problems are calculated with quadrature.

The solutions obtained are easily realized in the form of software products and take into account the constant slope of the axis of the gas pipeline, which is especially important when calculating pipelines with large diameters operating at high and ultra-high operating pressures.

Keywords: main gas pipeline, stationary and periodic operation modes, transition processes, mathematical modeling, telegraph equation, variables separation method, functional series

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1 Introduction

Pipeline networks designed to transfer the product (water, natural gas, petroleum, petroleum products, etc.) or mechanical energy over a certain distance are designed for a particular steady operation state. The established mode of operation can be stationary, when the indicators of the network and its links remain constant in time, or periodic. In other cases, there are transition processes that can be divided into the following types of tasks:

– the transition from one stationary state to another stationary state;
– the transition from a stationary state to a periodic state;
– the transition from a periodic state to a stationary state;
– the transition from one periodic state to another periodic state.

The repeated formation and propagation of shock waves and vacuum leads to the formation of zones of fatigue stresses in the pipeline network. The transition processes in pipelines are characterized by abrupt changes in hydrostatic pressure (for example, at starting and stopping processes of connected supercharger) or mass flow rate (for example, when a consumer is connected or disconnected in a radial network or a parallel-connected power unit). In this regard, it is advised to formulate and solve problems involving functional series (for example, Fourier series) in time and distance. Moreover, the formation of aperiodic solutions, due to both the intrinsic frequencies of the pipeline and the frequencies of external disturbances, can be expected. The eigenfrequencies of the elementary section are determined by a period that is equal to the ratio of the length of the section to the velocity of propagation of small pressure disturbances (sound velocity). And the period of external disturbances can be arbitrary.

Another reason for addressing to the transition problems in pipelines in general, and in main gas pipelines in particular, is the deviation of the network's indicators from the intended technological indicators due to the dynamic change in flow. To eliminate these deviations, you must first study their nature in theoretical terms. In mathematical modeling of the functioning of pipelines within the framework of a quasi-one-dimensional approach, all these transition processes are described by a system of equations of Zhukovsky N.E., which is usually supplemented by power factors, as well as the equations of conservation of energy and gas state. A more adequate mathematical model of the process of pipeline gas transportation is presented in the monograph [1], which takes into account almost all power and energy factors affecting the transition processes.

According to [2], partial or complete blockage in the main pipelines for the transportation of natural gas leads to a low efficiency of the entire system, and increases the risk of a potential accident. Therefore, effective detection and real-time monitoring of partial blockages (blockages) are important to ensure the safe operation of the pipeline. Since natural gas pipelines are a closed hydrodynamic system, a decrease in pipeline capacity or pipeline shutdown directly affects upstream and downstream users, resulting in economic losses for both pipeline operators and consumers. Based on the analytical solution method, this article presents a method for detecting partial blockage for natural gas pipelines under normal operating conditions. A partial blocking model was used and an analytical solution was obtained for the direct problem of inverting partial blocking parameters by mass flow with allowance for random errors. It was used as observable values in the implementation of Tikhonov's regularization method in order to establish an objective optimization function that contained partial blocking parameters and random measurement errors. The genetic algorithm is used to solve the inverse problem of parameter identification. Numerous computational experiments have shown that the proposed theoretical method can be used to identify the parameters of partial blocking of a gas pipeline with an extended partial overlap for a pipeline under normal operating conditions, where the observed parameters contain random errors.

In the work of Kim A.S., Mansurov M.N. [3], a methodology is proposed for selecting and calculating a pipeline system that ensures the fulfillment of the transport-consumer task during the development of offshore hydrocarbon fields, taking into account the reliability of underwater pipelines. The optimal transport scheme is determined by production rates, the number of platforms, their distance from the coast and from each other, sea depths along the pipeline routes, as well as economic indicators characterizing the cost of hydrocarbons and the laying of pipelines. Pipeline transitions from one state to another are described by terms of graph theory, which provide a visual representation of the nature of this process and simplify the procedure for compiling and solving Erlang equations.

The thesis of Yermolayeva N.N. [4] presents mathematical models of unsteady turbulent flows of a multicomponent gas mixture at ultrahigh pressures along the offshore gas pipelines, taking into account heat exchange through a multilayer wall. External tasks of the Stefan type of external freezing of the pipeline in the conditions of the northern seas are considered and new models and methods for solving problems based on these models are proposed. For the process of gas transportation through the offshore gas pipeline under freezing conditions, a mathematical model and a method for numerical solution of the corresponding problems with variable relief have been proposed.

The article [5] considers the issue for building a high-performance computing system for solving the problem of modeling non-stationary operating modes of gas pipelines. The authors propose the use of GRID technologies using asynchronous iterative methods. The examples show the effectiveness of asynchronous iterative methods for solving systems of linear algebraic equations.

In the work of Panferov V.I. and Panferov S.V. [6] two problems of modeling transition processes in gas supply systems are considered. The formulation of the problem considers the general form of the system of equations. In the first case, the transition process on a long pipeline is considered, when the loss caused by friction is large enough, in the second case, the loss due to friction is less enough or the pipeline is short.

The problems are brought to the solution of parabolic and hyperbolic equations with respect to pressure. The pressure at the beginning of the section is assumed to be constant, and at the end of the section, due to the closing of the valve, the flow rate and its consumption rate are zero. An analytical solution was obtained by the method of separating variables and graphs of pressure changes are given. From the pressure graphs it follows that the transition process is aperiodic in nature, the pressure rises smoothly to the required pressure level at all points of the pipeline. The graphs of the solution show the pressure distribution along the length of the pipeline at different points of time, when the phenomenon is described by the wave equation.

Follows from the conclusion of work that the real transition process caused by closing of the valve at the end of the gas pipeline, obviously has an appearance of decaying oscillations, the largest vibration amplitude is observed directly at the valve, however pressure increment is not as big as it takes place at a hydraulic shock in a dropping liquid.

Different factors of pipeline transportation of liquid and gas environments are analyzed in works [7–10].

With regard to the power factors taken into account and the methods of solving problems, this work is close to [11–14]. Introduction of gas mass flow and linearization of I.E. Charny allows us to make separate equations for hydrostatic pressure and mass flow, which are the equations of telegraph type. The problems are solved by the Fourier method involving functional series. The advantages of these solutions lie in the fact that the selection of the approaches of the “long” and “short” pipelines is made as if automatically, i.e. according to the results.

A similar approach was used in [15], where the problem was solved for given periodic boundary conditions of mass flow and gas pressure.

When considering the four types of tasks listed above, we limit ourselves to the cases when the values of one or another indicator are given at the boundaries. At the same time, if the mass flow values are set at the boundaries, then the problem with respect to pressure has boundary conditions of the third kind. Accordingly, if pressure values are set at the boundaries, then, when solving the problem with respect to mass flow, we have another kind of boundary conditions. In this regard, when setting a fixed indicator at the border, to determine another indicator, we use the already known solution of the indicator, the boundary conditions of which are set.

Despite this, the expected solution summarizes the results of a number of problems on the gas-dynamic state of the elementary section of the gas pipeline during transient processes, including those from [6, 11].

2 Quasi-one-dimensional equations of real gas pipeline transport used in the modeling of transition processes

For the description of transients occurring in the area with a constant slope and constant diameter D, we use the equations [1]

$$\begin{cases} -\dfrac{\partial p}{\partial x} = \dfrac{\lambda w^2}{2D}\rho + \rho g \sin\alpha + \dfrac{\partial \rho w}{\partial t}, \\ -\dfrac{1}{c^2}\dfrac{\partial p}{\partial t} = \dfrac{\partial(\rho w)}{\partial x}, \quad p = Z\rho RT. \end{cases} \tag{1}$$

Hereinafter, the variables are pressure p, density ρ and velocity w, the values of which are averaged over the cross-sectional area $f = \pi D^2/4$ and depend on the coordinate x and time t. The acceleration of gravity g, the velocity of propagation of small perturbations of pressure (sound) in the gas-pipe system c, the gas super-compressibility coefficient Z, the reduced gas constant R and the temperature T of the transported gas have constant or averaged values.

In three steps, we simplify the equations of system (1). In the first step in the first two equations, the density ρ is replaced by the expression p found from the third equation. In the second step, we introduce the linearization of a member of the Darcy-Weisbach law by replacing $w^2 \approx w_* w$, where $w_* = const$ is the characteristic speed of the process or the linearization parameter. In the third step, we introduce the mass flow

$$M = \rho w f.$$

As a result of these steps, the system of equations becomes linear:

$$\begin{cases} -\dfrac{\partial p}{\partial x} = \dfrac{b}{f}M + \dfrac{a}{c^2}p + \dfrac{1}{f}\dfrac{\partial M}{\partial t}, \\ -\dfrac{\partial p}{\partial t} = \dfrac{c^2}{f}\dfrac{\partial M}{\partial x}. \end{cases} \tag{2}$$

In the first equation, the term with the coefficient $a = \dfrac{g\sin\alpha}{ZRT}c^2$ reflects the force of gravity, the term with the coefficient $b = \dfrac{\lambda w_*}{2D}$ represents the friction force, and the third term to the right of the equal sign indicates the fraction of the local component of the gas inertia force in the pressure drop. According to the physics of the problem b has a non-negative value, and can have positive, zero or negative values.

With the exclusion of pressure p from system (2), we obtain an equation for mass flow M [12]:

$$\frac{\partial^2 M}{\partial t^2} + b\frac{\partial M}{\partial t} = c^2\frac{\partial^2 M}{\partial x^2} + a\frac{\partial p}{\partial x}, \tag{3}$$

and with the exception of mass flow M from (3) we obtain the equation for hydrostatic pressure p:

$$\frac{\partial^2 p}{\partial t^2} + b\frac{\partial p}{\partial t} = c^2\frac{\partial^2 p}{\partial x^2} + a\frac{\partial p}{\partial x}. \tag{4}$$

3 Statement of a general problem in terms of mass flow

We suppose that before the start of changes at the ends of the section, the distribution of the mass flow rate and its time derivative at the site are known:

$$M\left(x,0\right) = \varphi\left(x\right), \ \frac{\partial M(x,0)}{\partial t} = \psi(x) \text{ when } 0 \leqslant x \leqslant l, \ t < 0. \tag{5}$$

Starting from the moment of time $t = 0$ at the beginning ($x = 0$) and end ($x = l$) of the section, certain laws of changing the mass flow rate of gas will be established:

$$M(0,t) = M_0(t), \ M(l,t) = M_l(t) \text{ when } t \geqslant 0. \tag{6}$$

Such changes occur when consumers are turned on and/or disconnected, who are connected to the pipeline network before and after the sections under consideration. The reason for the change in mass flow rate at the inlet of the section may be the start-up and shutdown of the supercharger (compressor) connected in parallel to the other superchargers.

The functions $\varphi(x)$, $\psi(x)$, $M_0(t)$ and $M_l(t)$ can be continuous, piecewise continuous functions of their argument or constant. Functions $M_0(t)$ and $M_l(t)$ must be twice differentiable functions. At the points $(0,0)$ and $(l,0)$ functions expressing the initial and boundary conditions may have different values, which lead to the discontinuity of the desired functions at these points.

4 The solution of the problem (3), (5) - (6)

To solve the problem, we use the method of separation of variables [11, 16].

In order to facilitate the process of solving equation (3), a new unknown $u(x,t)$ is introduced according to

$$M(x,t) = e^{-\frac{bt}{2} - \frac{ax}{2c^2}} u(x,t). \tag{7}$$

In this case, the equation becomes simpler:

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} + \frac{b^2c^2 - a^2}{4c^2}u. \tag{8}$$

The boundary conditions for this equation are:

$$u\left(x,0\right) = e^{\frac{a}{2c^2}x}\varphi\left(x\right), \ \frac{\partial u\left(x,0\right)}{\partial t} - \frac{b}{2}u\left(x,0\right) = e^{\frac{a}{2c^2}x}\psi\left(x\right), \tag{9}$$

$$u\left(0,t\right) = e^{\frac{bt}{2}}M_0\left(t\right), \ u\left(l,t\right) = e^{\frac{bt}{2} + \frac{al}{2c^2}}M_l\left(t\right). \tag{10}$$

It is possible to prove that with the introduction of a new function [$v(x,t)$ according to

$$u\left(x,t\right) = v\left(x,t\right) + e^{\frac{bt}{2}}M_0\left(t\right) + \frac{x}{l}e^{\frac{bt}{2}}\left[e^{\frac{al}{2c^2}}M_l\left(t\right) - M_0\left(t\right)\right] \tag{11}$$

boundary conditions become uniform

$$v(0,t) = v(l,t) = 0$$

and the task is reduced to a form that allows the use of the method of separation of variables.

In this case, the equation becomes inhomogeneous:

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} + \frac{b^2c^2 - a^2}{4c^2} v + F(t) + G(t)x,$$

where

$$F\left(t\right) = -e^{\frac{bt}{2}} \left[M''_0\left(t\right) + bM'_0\left(t\right) + \frac{a^2}{4c^2} M_0\left(t\right) \right],$$

$$G\left(t\right) = -\frac{1}{l} e^{\frac{bt}{2}} \left\{ e^{\frac{al}{2c^2}} \left[M''_l\left(t\right) + bM'_l\left(t\right) + \frac{a^2}{4c^2} M_l\left(t\right) \right] - \right.$$
$$\left. - \left[M''_0\left(t\right) + bM'_0\left(t\right) + \frac{a^2}{4c^2} M_0\left(t\right) \right] \right\}.$$

We look for the solution $v(x,t)$ of the problem in the form of a sum:

$$v(x,t) = U(x,t) + V(x,t).$$

The first term $U(x,t)$ represents the general solution of a homogeneous equation

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2} + \frac{b^2c^2 - a^2}{4c^2} U \tag{12}$$

under zero boundary conditions

$$U(0,t) = U(l,t) = 0. \tag{13}$$

The second term $V(x,t)$ is a particular solution of the inhomogeneous equation

$$\frac{\partial^2 V}{\partial t^2} = c^2 \frac{\partial^2 V}{\partial x^2} + \frac{b^2c^2 - a^2}{4c^2} V + F(t) + G(t)x \tag{14}$$

also under zero boundary conditions

$$V(0,t) = V(l,t) = 0.$$

We will return to the initial conditions after determining all the necessary components $u(x,t)$.

The use of the method of separation of variables [16] to equation (12) with boundary conditions (13) leads to the solution:

$$U(x,t) = \sum_{n=1}^{\infty} Y_n(t) X_n(x),$$

where the eigenfunctions are

$$X_n(x) = sin \frac{\pi n x}{l},$$

$$Y_n(t) = \begin{cases} C_n ch\sqrt{\mathfrak{D}_n}t + D_n sh\sqrt{\mathfrak{D}_n}t \text{ when } \mathfrak{D}_n > 0, \\ C_n + D_n t \text{ when } \mathfrak{D}_n = 0, \\ C_n \cos\sqrt{|\mathfrak{D}_n|}t + D_n \sin\sqrt{|\mathfrak{D}_n|}t \text{ when } \mathfrak{D}_n < 0. \end{cases}$$

Here $\mathfrak{D}_n = \dfrac{b^2c^2 - a^2}{4c^2} - c^2\lambda_n^2$, $\lambda_n = \dfrac{\pi n}{l}$.

The square of the norm of the eigenfunctions $X_n(x)$ is $\|X^2(x)\| = l/2$.

The final form of the general solution of equation (12) with zero boundary conditions is

$$U(x,t) = \sum_{n=1}^{\infty} \begin{pmatrix} C_n ch\sqrt{\mathfrak{D}_n}t + D_n sh\sqrt{\mathfrak{D}_n}t \text{ when } \mathfrak{D}_n > 0 \\ C_n + D_n t \quad \mathfrak{D}_n = 0 \\ C_n \cos\sqrt{|\mathfrak{D}_n|}t + D_n \sin\sqrt{|\mathfrak{D}_n|}t \quad \mathfrak{D}_n < 0 \end{pmatrix} \sin\frac{\pi n x}{l}. \tag{15}$$

The particular solution of the inhomogeneous equation (14), with zero boundary conditions, we look for in the form

$$V(x,t) = \sum_{n=1}^{\infty} L_n(t) \sin\frac{\pi n x}{l}. \tag{16}$$

The statement of this solution in (14) leads to a second-order differential equation

$$L''_n(t) - \mathfrak{D}_n L_n(t) = \frac{2}{\pi n}\left[1 - (-1)^n\right] F(t) - (-1)^n \frac{2l}{\pi n} Q(t), \tag{17}$$

where the following factorizations are

$$1 = \sum_{n=1}^{\infty} q_n \sin\frac{\pi n x}{l},\ x = \sum_{n=1}^{\infty} g_n \sin\frac{\pi n x}{l}, \tag{18}$$

when $q_n = \dfrac{2}{\pi n}\left(1 - (-1)^n\right)$ and $g_n = -(-1)^n \dfrac{2l}{\pi n}$.

With constant, periodic, and some other types of functions $F(t)$ and $G(t)$, equation (17) is easily solved. We consider that this equation is solved and the value of the function is determined $L_n(t)$.

Thus, we got a solution for $v(x,t)$:

$$v(x,t) = U(x,t) + V(x,t) = \sum_{n=1}^{\infty} L_n(t) \sin\frac{\pi n x}{l} +$$
$$+ \sum_{n=1}^{\infty} \begin{pmatrix} C_n ch\sqrt{\mathfrak{D}_n}t + D_n sh\sqrt{\mathfrak{D}_n}t \text{ when } \mathfrak{D}_n > 0 \\ C_n + D_n t \text{ when } \mathfrak{D}_n = 0 \\ C_n \cos\sqrt{|\mathfrak{D}_n|}t + D_n \sin\sqrt{|\mathfrak{D}_n|}t \text{ when } \mathfrak{D}_n < 0 \end{pmatrix} \sin\frac{\pi n x}{l}.$$

Reverse transition to the solution of equation (8) $u(x,t)$ we will produce according to (11):

$$u\left(x,t\right)=e^{\frac{bt}{2}}M_{0}\left(t\right)+\frac{x}{l}e^{\frac{bt}{2}}\left[e^{\frac{al}{2c^{2}}}M_{l}\left(t\right)-M_{0}\left(t\right)\right]+\sum_{n=1}^{\infty}L_{n}\left(t\right)\sin\frac{\pi nx}{l}+$$
$$+\sum_{n=1}^{\infty}\left(\begin{array}{l}C_{n}ch\sqrt{\mathfrak{D}_{n}}t+D_{n}sh\sqrt{\mathfrak{D}_{n}}t\ \text{ when }\ \mathfrak{D}_{n}>0\\ C_{n}+D_{n}t\quad\text{when}\ \ \mathfrak{D}_{n}=0\\ C_{n}\cos\sqrt{\left|\mathfrak{D}_{n}\right|}t+D_{n}\sin\sqrt{\left|\mathfrak{D}_{n}\right|}t\ \text{ when }\ \mathfrak{D}_{n}<0\end{array}\right)\sin\frac{\pi nx}{l}.$$

To find the values of the unknown coefficients, we implement the initial conditions (9).

The use of the orthonormality of eigenfunctions $X_n(x)$ [16], expansions (18) and the values of the integral

$$I_{n}=\int_{0}^{l}e^{\frac{a\xi}{2c^{2}}}\varphi(\xi)\sin\frac{\pi n\xi}{l}d\xi \tag{19}$$

let us find the value of C_n from the first condition (9):

$$C_{n}=-M_{0}(0)q_{n}-\frac{1}{l}\left[e^{\frac{al}{2c^{2}}}M_{l}(0)-M_{0}(0)\right]g_{n}-L_{n}(0)+\frac{2}{l}I_{n}.$$

The second initial condition from (9), after calculating and substituting the values of $u(x,0)$ and $\dfrac{\partial u(x,0)}{\partial t}$, leads to the value of the coefficients

$$D_{n}=\frac{1}{\gamma_{n}}\left\{-L'_{n}\left(0\right)+\frac{b}{2}L_{n}\left(0\right)+\frac{b}{2}C_{n}-M'_{0}\left(0\right)q_{n}-\right.$$
$$\left.-\frac{1}{l}\left[e^{\frac{al}{2c^{2}}}M'_{l}\left(0\right)-M'_{0}\left(0\right)\right]g_{n}+J_{n}\right\}.$$

Here

$$J_{n}=\int_{0}^{l}e^{\frac{ax}{2c^{2}}}\psi(\xi)\sin\frac{\pi n\xi}{l}d\xi, \tag{20}$$

$$\gamma_{n}=\left(\begin{array}{l}\sqrt{\mathfrak{D}_{n}}\text{ when }\mathfrak{D}_{n}>0\\ 1\text{ when }\mathfrak{D}_{n}=0\\ \sqrt{\left|\mathfrak{D}_{n}\right|}D_{n}\text{ when }\mathfrak{D}_{n}<0\end{array}\right).$$

The reverse transition to mass consumption in (7) gives the final form of the solution to problem (3), (5) - (6) with respect to mass consumption:

$$M(x,t) = e^{-\frac{ax}{2c^2}} M_0(t) + \frac{x}{l} e^{-\frac{ax}{2c^2}} \left[e^{\frac{al}{2c^2}} M_l(t) - M_0(t) \right] +$$
$$+ e^{-\frac{bt}{2} - \frac{ax}{2c^2}} \sum_{n=1}^{\infty} \left[\begin{pmatrix} C_n ch\sqrt{\mathfrak{D}_n}t + D_n sh\sqrt{\mathfrak{D}_n}t \text{ when } \mathfrak{D}_n > 0 \\ C_n + D_n t \text{ when } \mathfrak{D}_n = 0 \\ C_n \cos\sqrt{|\mathfrak{D}_n|}t + D_n \sin\sqrt{|\mathfrak{D}_n|}t \text{ when } \mathfrak{D}_n < 0 \\ \end{pmatrix} + L_n(t) \right] \times \quad (21)$$
$$\times \sin\frac{\pi n x}{l}.$$

5 The solution of the problem of pressure using (21)

The solution of the problem with respect to pressure can be obtained by solving the equation (4) under initial and boundary conditions, which are formed from (5) and (6) according to the equations of system (2). The resulting boundary conditions will belong to the third type. Accordingly, the whole process of solving the problem is repeated, that means, it is necessary to find eigenvalues, eigenfunctions for x and t, to prove the orthonormality of eigenfunctions for x and so on. To avoid this complex process of solving the problem with respect to pressure, we turn directly to the system of equations (2) and the resulting expression (21).

The second equation of system (2) is written in the form of $\frac{\partial p}{\partial t} = -\frac{c^2}{f}\frac{\partial M}{\partial x}$ and we integrate both sides of the equation in time from zero to t:

$$p(x,t) = p(x,0) - \frac{c^2}{f}\int_0^t \frac{\partial M(x,\xi)}{\partial x} d\xi. \quad (22)$$

The expression of the initial pressure distribution $p(x,0)$ is found from the first equation of the system (2). The values $M(x,0)$ and $\frac{\partial M(x,0)}{\partial t}$ are known for $0 \leqslant x \leqslant l$ and $t > 0$. In this regard, when $t \to 0$ the first equation of system (2) can be written in the form:

$$\frac{\partial p}{\partial x} + \frac{a}{c^2}p = -\frac{b\varphi(x) + \psi(x)}{f}.$$

Multiplying both sides of the equation by $e^{\frac{ax}{c^2}}$, its left-hand side can be represented as a monomial:

$$\frac{\partial(e^{\frac{ax}{c^2}}p)}{\partial x} = -e^{\frac{ax}{c^2}}\frac{b\varphi(x) + \psi(x)}{f}.$$

Now, supposing $p(0,0) = p_{00}$ we integrate the equation from zero to x:

$$e^{\frac{ax}{c^2}}p(x,0) - p_{00} = \Phi(x),$$

where

$$\Phi(x) = -\frac{b}{f}\int_0^x e^{\frac{a\eta}{c^2}}\varphi(\eta)d\eta - \frac{1}{f}\int_0^x e^{\frac{a\eta}{c^2}}\psi(\eta)d\eta.$$

Hence we find that, according to the initial conditions (5), the initial distribution of pressure over the section can be described by the formula

$$p(x,0) = e^{-\frac{ax}{c^2}}\left[p_{00} + \Phi\left(x\right)\right].$$

The integral is involved in (22). First we find the integrand function:

$$\frac{\partial M\left(x,\xi\right)}{\partial x} = -\frac{a}{2c^2}e^{-\frac{ax}{2c^2}}M_0\left(\xi\right) +$$
$$+\frac{1}{l}e^{-\frac{ax}{2c^2}}\left(1-\frac{a}{2c^2}x\right)\left[e^{\frac{al}{2c^2}}M_l\left(\xi\right) - M_0\left(\xi\right)\right] +$$
$$+e^{-\frac{b\xi}{2}-\frac{ax}{2c^2}}\sum_{n=1}^{\infty}\left[Y_n\left(\xi\right)+L_n\left(\xi\right)\right]\left(\frac{\pi n}{l}\cos\frac{\pi n x}{l} - \frac{a}{2c^2}\sin\frac{\pi n x}{l}\right).$$

The integration of this expression concerns parts that depend on time, and the remaining factors will appear as coefficients of integration.

Then we select the parts that depend on time, and integrate them:

$$\bar{M}_0\left(t\right) = \int\limits_0^t M_0\left(\xi\right)d\xi,\ \ \bar{M}_l\left(t\right) = \int\limits_0^t M_l\left(\xi\right)d\xi,$$
$$\bar{L}_n\left(t\right) = \int\limits_0^t e^{-\frac{b\xi}{2}}L_n\left(\xi\right)d\xi,\ \ \bar{Y}_n\left(t\right) = \int\limits_0^t e^{-\frac{b\xi}{2}}Y_n\left(\xi\right)d\xi.$$

The first three integrals depend on the functions $M_0(t)$ and $M_l(t)$. They can be calculated analytically or numerically. The fourth integral is obviously independent of the boundary conditions, but is defined as a conditional operator depending on the value of the expression $\mathfrak{D}_n$. The value of the integral is taken from [11].

When $\mathfrak{D}_n > 0$ only time dependent part has integral

$$\bar{Y}_n^{(1)}\left(t\right) = \int\limits_0^t e^{-\frac{b\xi}{2}}\left(C_n ch\sqrt{\mathfrak{D}_n}\xi + D_n sh\sqrt{\mathfrak{D}_n}\xi\right)d\xi =$$
$$= \frac{1}{\frac{a^2}{4c^2}+c^2\lambda_n^2}\left[\left(-\frac{b}{2}C_n - \sqrt{\mathfrak{D}_n}D_n\right)\left(e^{-\frac{bt}{2}}ch\sqrt{\mathfrak{D}_n}t - 1\right) + \right.$$
$$\left. + \left(-\sqrt{\mathfrak{D}_n}C_n - \frac{b}{2}D_n\right)e^{-\frac{bt}{2}}sh\sqrt{\mathfrak{D}_n}t\right],$$

when $\mathfrak{D}_n = 0$ –

$$\bar{Y}_n^{(2)}\left(t\right) = \int\limits_0^t e^{-\frac{b\xi}{2}}\left(C_n + D_n t\right)d\xi =$$
$$= -\frac{2}{b}\left(e^{-\frac{bt}{2}}-1\right)C_n + \left[-\frac{2}{b}te^{-\frac{bt}{2}} - \frac{4}{b^2}\left(e^{-\frac{bt}{2}}-1\right)\right]D_n,$$

when $\mathfrak{D}_n < 0$ –

$$\bar{Y}_n^{(3)}(t) = \int_0^t e^{-\frac{b\xi}{2}} \left(C_n \cos\sqrt{|\mathfrak{D}_n|}\xi + D_n \sin\sqrt{|\mathfrak{D}_n|}\xi \right) d\xi =$$
$$= \frac{1}{\frac{a^2}{4c^2} + c^2\lambda_n^2} \left[\left(-\frac{b}{2}C_n - \sqrt{|\mathfrak{D}_n|}D_n \right) \left(e^{-\frac{bt}{2}} \cos\sqrt{|\mathfrak{D}_n|}t - 1 \right) + \right.$$
$$\left. + \left(\sqrt{|\mathfrak{D}_n|}C_n - \frac{b}{2}D_n \right) e^{-\frac{bt}{2}} \sin\sqrt{|\mathfrak{D}_n|}t \right].$$

Substituting the obtained integrals into equation (22) leads to the solution of the problem with respect to pressure:

$$p(x,t) = e^{-\frac{ax}{c^2}} \left[p_{00} + \Phi(x) \right] -$$
$$-\frac{c^2}{f} e^{-\frac{ax}{2c^2}} \left\{ -\frac{a}{2c^2} \bar{M}_0(t) + \frac{1}{l}\left(1 - \frac{a}{2c^2}x\right) \left[e^{\frac{al}{2c^2}} \bar{M}_l(t) - \bar{M}_0(t) \right] + \right. \tag{23}$$
$$\left. + \sum_{n=1}^{\infty} \left[\bar{Y}_n(t) + \bar{L}_n(t) \right] \left(\frac{\pi n}{l} \cos\frac{\pi n x}{l} - \frac{a}{2c^2} \sin\frac{\pi n x}{l} \right) \right\}.$$

Simultaneous participation $\sin\dfrac{\pi n x}{l}$ and $\cos\dfrac{\pi n x}{l}$ in the eigenfunction of the problem with respect to $p(x,t)$ with x (the parenthesis under the sum) indicates the participation in the boundary conditions $p(x,t)$ for both the desired function and its derivative with respect to x, that is, boundary conditions of the third type.

6 The solution of the problem relative to pressure when setting the boundary conditions for pressure

In this case, the boundary conditions are

$$p(x,0) = \varphi(x), \ \frac{\partial p(x,0)}{\partial t} = \psi(x) \ \text{ for } 0 \leqslant x \leqslant l, \ t < 0; \tag{24}$$

$$p(0,t) = p_0(t), \ p(l,t) = p_l(t) \ \text{ for } t \geqslant 0. \tag{25}$$

The solution to this problem, according to the already existing solution regarding mass flow, is written in the form:

$$p(x,t) = e^{-\frac{ax}{2c^2}} p_0(t) + \frac{x}{l} e^{-\frac{ax}{2c^2}} \left[e^{\frac{al}{2c^2}} p_l(t) - p_0(t) \right] +$$
$$+ e^{-\frac{bt}{2} - \frac{ax}{2c^2}} \sum_{n=1}^{\infty} \left[\left(\begin{array}{l} A_n ch\sqrt{\mathfrak{D}_n}t + B_n sh\sqrt{\mathfrak{D}_n}t \ \text{ when } \ \mathfrak{D}_n > 0 \\ A_n + B_n t \ \text{ when } \ \mathfrak{D}_n = 0 \\ A_n \cos\sqrt{|\mathfrak{D}_n|}t + B_n \sin\sqrt{|\mathfrak{D}_n|}t \ \text{ when } \ \mathfrak{D}_n < 0 \end{array} \right) + N_n(t) \right] + \tag{26}$$
$$+ \sin\tfrac{\pi n x}{l}.$$

The values of the coefficients A_n, B_n and the function $N_n(t)$, that are entered instead of C_n, D_n and $L_n(t)$ are determined as it was done in solving the problem of mass flow, but using conditions (24) and (25).

7 The solution of the mass flow problem when the boundary conditions for pressure are given

In this case, it is advisable to refer directly to the system of equations (2), and not to equation (3).

When $t = 0$ the condition $\dfrac{\partial p(x,0)}{\partial t} = \psi(x)$ is known. Substituting it into the second equation of system (2), and taking $M(0,0) = M_{00}$, we obtain the initial distribution of mass flow according to conditions (24):

$$M(x,0) = M_{00} - \frac{f}{c^2}\int\limits_0^x \psi(\xi)d\xi.$$

The first equation of system (2) is written as

$$\frac{\partial}{\partial t}\left(e^{bt}M\right) = -fe^{bt-\frac{ax}{c^2}}\frac{\partial(e^{\frac{ax}{c^2}}p)}{\partial x}.$$

We integrate both sides of this equality from zero to t:

$$e^{bt}M\left(x,t\right) - M\left(x,0\right) = -fe^{-\frac{ax}{c^2}}\int\limits_0^t \frac{\partial e^{\frac{ax}{c^2}+b\eta}p\left(x,\eta\right)}{\partial x}d\eta.$$

From here we find the expression for mass flow:

$$M\left(x,t\right) = e^{-bt}M\left(x,0\right) - fe^{-bt-\frac{ax}{c^2}}\int\limits_0^t \frac{\partial e^{\frac{ax}{c^2}+b\eta}p\left(x,\eta\right)}{\partial x}d\eta. \tag{27}$$

Find the integrand function:

$$\frac{\partial e^{\frac{ax}{c^2}+b\eta}p\left(x,\eta\right)}{\partial x} =$$

$$= \frac{a}{2c^2}e^{\frac{ax}{2c^2}+b\eta}p_0\left(\eta\right) + \frac{1}{l}\left(1+\frac{a}{2c^2}x\right)e^{\frac{ax}{2c^2}+b\eta}\left[e^{\frac{al}{2c^2}}p_l\left(\eta\right) - p_0\left(\eta\right)\right] +$$

$$+\sum_{n=1}^{\infty}\left[\left(\begin{array}{l} A_n ch\sqrt{\mathfrak{D}_n}\eta + B_n sh\sqrt{\mathfrak{D}_n}\eta \ \text{ when } \ \mathfrak{D}_n > 0 \\ A_n + B_n\eta \quad \text{when } \ \mathfrak{D}_n = 0 \\ A_n\cos\sqrt{|\mathfrak{D}_n|}\eta + B_n\sin\sqrt{|\mathfrak{D}_n|}\eta \ \text{ when } \ \mathfrak{D}_n < 0 \end{array}\right) + N_n\left(\eta\right)\right]\times$$

$$\times e^{\frac{b\eta}{2}+\frac{ax}{2c^2}}\left(\frac{a}{2c^2}\sin\frac{\pi nx}{l} + \frac{\pi n}{l}cos\frac{\pi nx}{l}\right).$$

First, we calculate the integrals:

$$\bar{p}_0\left(t\right) = \int\limits_0^t e^{b\eta}p_0\left(\eta\right)d\eta,\ \ \bar{p}_l\left(t\right) = \int\limits_0^t e^{b\eta}p_l\left(\eta\right)d\eta,$$

$$\bar{N}_n\left(t\right) = \int\limits_0^t e^{\frac{b\eta}{2}}N_n\left(\eta\right)d\eta,\ \ \hat{Y}_n\left(t\right) = \int\limits_0^t e^{\frac{b\eta}{2}}\tilde{Y}_n\left(\eta\right)d\eta.$$

We assume that the values of the first three integrals are calculated according to the boundary conditions. We assume that the integral values of $\hat{Y}_n(t)$ are known: only in the formulas of $\bar{Y}_n(t)$ are the coefficients C_n and D_n should be replaced by A_n and B_n.

Substituting the obtained expressions for and the values of the integrals in (27) allows us to write the final solution for mass flow rate in the form:

$$M\left(x,t\right)=e^{-bt}\left[M_{00}-\frac{f}{c^2}\int\limits_0^x\psi\left(\zeta\right)d\zeta\right]-$$
$$-fe^{-bt-\frac{ax}{2c^2}}\left\{\frac{a}{2c^2}\bar{p}_0\left(t\right)+\frac{1}{l}\left(1+\frac{a}{2c^2}x\right)\left[e^{\frac{al}{2c^2}}\bar{p}_l\left(t\right)-\bar{p}_0\left(t\right)\right]+\right. \tag{28}$$
$$\left.+\sum_{n=1}^{\infty}\left[\hat{Y}_n\left(t\right)+\bar{N}_n\left(t\right)\right]\left(\frac{a}{2c^2}\sin\frac{\pi nx}{l}+\frac{\pi n}{l}cos\frac{\pi nx}{l}\right)\right\}.$$

At known values of pressure and mass flow rate, the gas density is determined by the formula $\rho=\dfrac{p}{ZRT}$, and the flow rate is determined by the formula $w=\dfrac{M}{\rho f}$.

8 Computational experiment

A program has been compiled for the case when boundary conditions are specified with respect to mass flow in the form of constant values. The initial data are taken as $l=1\,km$, $w_0=13.24\,m/s$, $w_*=6,62\,m/s$, $p_{00}=5\,MPa$, $\sin\alpha=0.1$, $c=200\,m/s$, $\lambda=0.01$, $M_0=M_H=400\,kg/s$, $M_K=0\,kg/s$.

Graphs of mass flow, hydrostatic pressure, and gas velocity for dimensionless times 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1.0, 1.5, 2.0 and 2.5 correspond to the case are presented in Fig. 1-3.

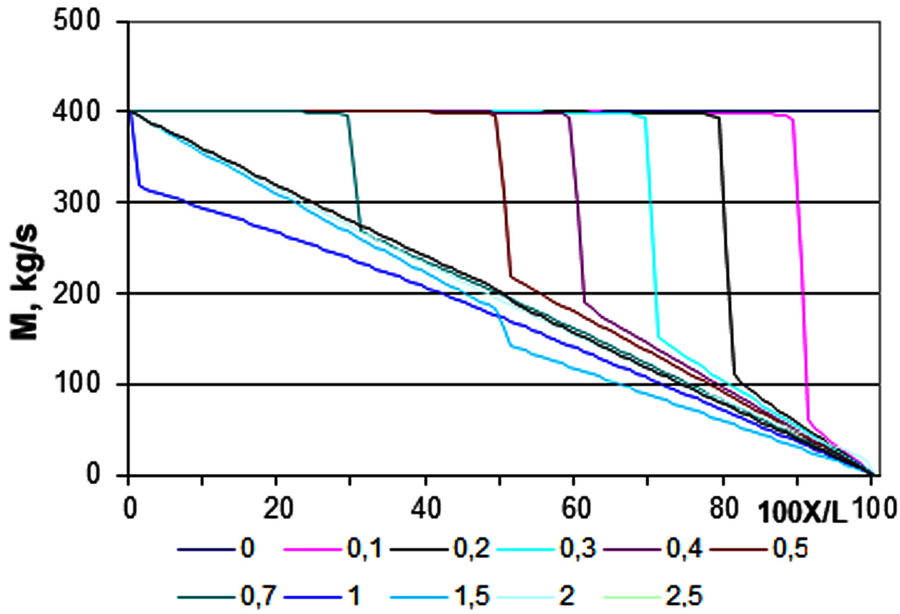

Figure 1 Graphs of mass flow at the exit from the site for various moments of dimensionless time. $l=1\,km$, $w_0=13.24\,m/s$, $w_*=6,62\,m/s$, $p_{00}=5\,MPa$, $\sin\alpha=0.1$, $c=200\,m/s$, $\lambda=0.01$, $M_0=M_H=400\,kg/s$, $M_K=0\,kg/s$.

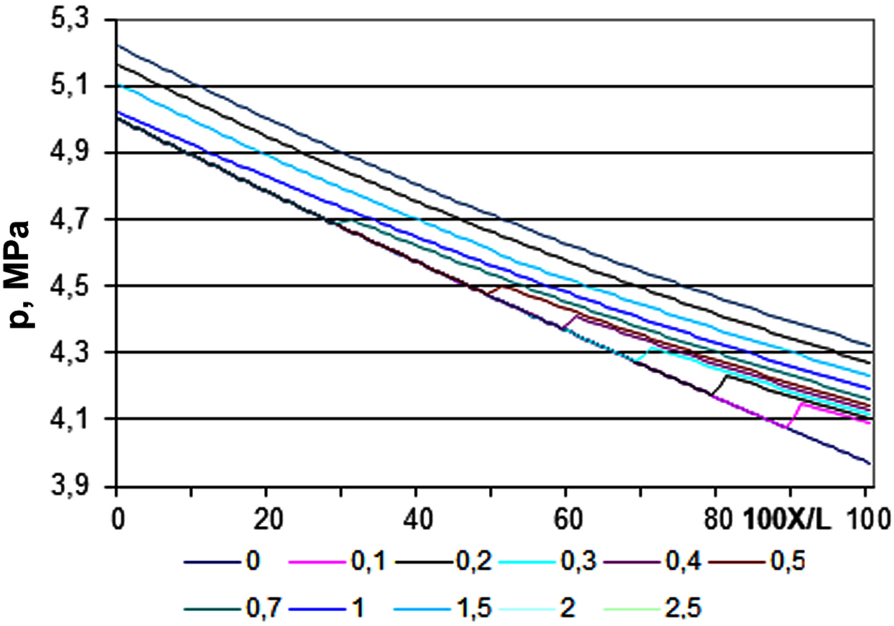

Figure 2 The distribution of hydrostatic pressure in the site at different points in time. Same data as in Fig. 1.

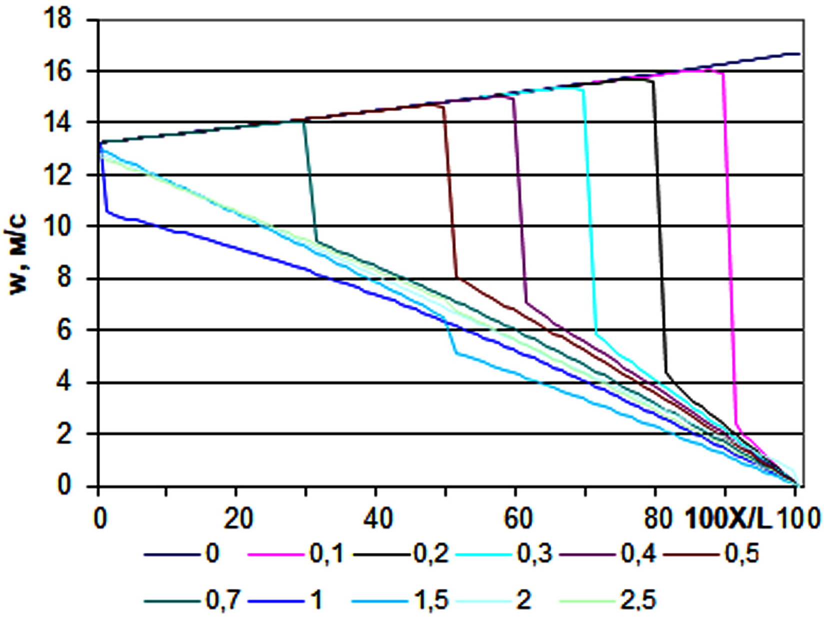

Figure 3 The change in the average flow rate in the site at different points in time. Same data as in Fig. 1.

Comparisons of the calculation results using the above formulas with the results of [11] showed their identity. This proves the reliability of the proposed solution for the first problem.

Works on the implementation of the solution method for the cases when the boundary conditions are specified in the form of Fourier series are carried out.

9 Conclusion

Taking into account the relevance of studying the characteristics of transition processes in gas pipelines, we formulated and analytically solved the problems for the cases of given boundary conditions with respect to mass flow and gas pressure. With the account of the possibility of the presence or formation of gaps in the boundary conditions, solutions were sought in the form of functional series. For cases of transition from a stationary or periodic mode of operation of the considered elementary segment to another stationary or periodic mode, the integrals given in the text are calculated in quadratures.

In the solutions obtained (21), (23), (26) and (28), the mechanisms of perturbation suppression due to the friction force are embedded. At the same time, the frequencies of disturbances, which are stored longer ($\mathfrak{D}_n < 0$), are highlighted, have a quasi-resonant nature ($\mathfrak{D}_n = 0$) or are quenched intensively ($\mathfrak{D}_n > 0$).

Taking into account the influence of gravity and the local component of inertia, as well as variable boundary conditions in a mathematical model of transition processes helps to ensure the adequacy of the solutions obtained for problems of pipeline gas transportation. And this corresponds to the trend of further development of the network of gas pipelines with the transition to large diameters and high working pressures.

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МОДЕЛИРОВАНИЕ ПЕРЕХОДНЫХ ПРОЦЕССОВ ПРИ ТРУБОПРОВОДНОЙ ТРАНСПОРТИРОВКЕ РЕАЛЬНЫХ ГАЗОВ

Хужаев И.К., Мамадалиев Х.А., Аминов Х.Х.
`husniddin_m1@bk.ru`
Научно-инновационный центр информационно-коммуникационных технологий,
100125 Узбекистан, Ташкент, Буз-2, 17А

Задачи идентификации утечки, участков накопления высоких напряжений и отклонения функции сети от технологических требований непосредственно связаны переходными процессами. В связи с этим исследование переходных процессов в трубопроводах является актуальным как с теоретической, так и с практической точки зрения. Математическое моделирование переходных процессов в магистральных трубопроводах производится в рамках квазиодномерных уравнений Н.Е. Жуковского с учетом сил трения, гравитации и инерции. Для газовой среды эти уравнения имеют третий порядок относительно неизвестных. Применением способа осреднения типа И.А. Чарного степень уравнений снижается на один порядок. А переход к массовому расходу, аналогу функции тока при решении двумерных уравнений гидродинамики, позволяет получить линейные уравнения относительно массового расхода, в выражении которых фигурирует произведение неизвестных плотности и скорости газа. Относительно массового расхода и давления составлены автономные уравнения, которые представляют тип телеграфного уравнения. С учетом возможных скачкообразных изменений искомых по времени и расстоянию решение ищется в виде функциональных рядов. Наглядность этого метода заключается в том, что выделяются частоты возмущений, свойственные уравнениям параболического и гиперболического типов, и промежуточного варианта. Представляется общий метод решения задач перехода от одного установившегося режима работы участка к другому установившемуся режиму работы. Под установившимся режимом подразумеваются стационарный и периодический режимы работы участка. В этих случая легко определяются производные и в квадратурах вычисляются интегралы, фигурирующие в общем решении задач. Полученные решения легко материализуются в виде программных продуктов и учитывают постоянный уклон оси газопровода, что особенно важно при расчете трубопроводов с большими диаметрами, функционирующих при высоких и сверхвысоких рабочих давлениях.

Ключевые слова: магистральный газопровод, стационарный и периодический режимы работы участка, переходные процессы, математическое моделирование, телеграфное уравнение, метод разделение переменных, функциональные ряды

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