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SIMULATION OF NON-ISOTHERMAL MULTIPHASE FLOW IN POROUS MEDIA USING EXPLICIT DIFFERENCE SCHEMES*

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The work deals with the development of algorithms and software for simulation of complex fluid flow in the subsurface. These tools can be used in important practical applications such as oil and gas recovery problems, ecological problems concerning the soil and groundwater contamination and many others. The major goal of the research is implementation of the governing model by explicit numerical methods with rather mild stability conditions in order to achieve high parallelization efficiency on modern supercomputers.

For this purpose a classical mathematical model of multiphase slightly compressible fluid flow in a porous medium has been modified by analogy with the quasigasdynamic system of equations, hyperbolization of the system has also been performed. Finally the phase continuity equation has got a regularizing term and the second time derivative with small parameters. The corresponding three-level explicit difference scheme has the second order of approximation in time and in space. The model takes into account possible heat sources, gravitational and capillary forces.

The proposed approach is verified by a number of test predictions, physically correct results are obtained numerically. High speed-up of computations is observed on hybrid clusters including multicore CPUs and accelerators like graphics processing units (GPUs).

Keywords: porous medium, multiphase fluid flow, quasigasdynamic system, explicit difference schemes, parallel computing

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1 Introduction

Mathematical modeling of multiphase fluid flows in porous media is necessary for solving practically important problems such as oil and gas recovery problems, ecological problems concerning the soil and groundwater contamination and many others. Models and algorithms for their solution still need improvement to predict processes in strata with sufficient accuracy at reasonable computing time [1].

Numerical simulation of these large-scale processes is very time-consuming and impossible without the employment of high-performance computing systems. Nowadays the

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rapid growth in the computer performance is mainly achieved due to the use of hybrid architectures including multicore CPUs and different accelerators like graphics processing units (GPU) [2]. However such architectures cause serious difficulties in the software development. Computational algorithms with logical simplicity, for example, explicit finite-difference schemes for the quasigasdynamic (QGD) system [3] can be adapted easily to supercomputers and allow to exploit them more efficiently.

Besides there is another reason of the interest in algorithms of the explicit type. Some oil recovery problems (problems with combustion fronts, phase transitions, complicated functions of the relative phase permeability) require calculations with very small space steps constraining time steps as $\Delta t = O\left(h^2\right)$ not only for explicit but also for implicit schemes. This is necessary in order to achieve the critical accuracy of the solution. Then explicit schemes can gain in terms of the total run time in comparison with implicit ones. An original approach to modeling porous medium flows is developed by the authors in accordance to this trend.

2 Mathematical model and numerical implementation

The work presents an original model of fluid flow through porous media constructed by analogy with the QGD system of equations [3] and allowing implementation via explicit numerical methods [4]. The derivation of kinetically-consistent finite difference schemes and the related QGD system is based on the so-called principle of minimum sizes in continuum mechanics . We assume the existence of minimal space and time scales which act as lower limits for description details [5]. In porous media the minimal reference length l is a distance at which rock microstructure is negligible (l is of the order of hundred rock grain sizes), the minimal reference time τ is the time needed to reach inner equilibrium in the volume of linear size l. Starting from the basic flow equations for slightly compressible fluid [6–8], using the minimal scales and the differential approximation technique the next multiphase flow model has been derived in the non-isothermal case (phase $\alpha = w$ (water), n (Non-Aqueous Phase Liquid – NAPL), g (gas); r denotes a rock):

$$\varphi \frac{\partial \left(\rho_\alpha S_\alpha\right)}{\partial t} + \tau \frac{\partial^2 \left(\rho_\alpha S_\alpha\right)}{\partial t^2} + \operatorname{div}\left(\rho_\alpha \mathbf{u}_\alpha\right) = q_\alpha + \operatorname{div} \frac{l c_\alpha}{2} \operatorname{grad}\left(\rho_\alpha S_\alpha\right), \tag{1}$$

$$\mathbf{u}_\alpha = -K \frac{k_\alpha}{\mu_\alpha} \left(\operatorname{grad} P_\alpha - \rho_\alpha \mathbf{g}\right), \tag{2}$$

$$\begin{aligned} \frac{\partial}{\partial t}\left[\varphi \sum_\alpha \rho_\alpha S_\alpha E_\alpha + (1-\varphi)\rho_r E_r\right] + \operatorname{div}\left(\sum_\alpha \rho_\alpha H_\alpha \mathbf{u}_\alpha\right) = \\ = \operatorname{div} \lambda_{\text{eff}} \operatorname{grad} T + \sum_\alpha \operatorname{div} \frac{l c_\alpha}{2} \rho_\alpha \operatorname{grad} T, \end{aligned} \tag{3}$$

$$\rho_g = \rho_{0g} \frac{P_g}{P_{0g}} \frac{T_0}{T}, \qquad \rho_\alpha = \rho_{0\alpha}\left[1 + \beta_\alpha \left(P_\alpha - P_{0\alpha}\right) - \eta_\alpha \left(T - T_0\right)\right], \qquad \alpha = w, n, \tag{4}$$

$$\sum_\alpha S_\alpha = 1. \tag{5}$$

Here S_α is the saturation, P_α is the pressure, ρ_α is the density, $\mathbf{u}_\alpha$ is the Darcy velocity, T is the temperature, E_α is the internal energy, q_α is the source of fluid, φ is the porosity, K is the absolute permeability, k_α is the relative phase permeability, μ_α is the dynamic viscosity, $\mathbf{g}$ is the gravity vector, c_α is the sound speed in α-phase, β_α is the coefficient of isothermal compressibility, η_α is the coefficient of thermal expansion, $P_{0\alpha}$, $\rho_{0\alpha}$ and T_0 are reference values, $\rho_r = const$ – the rock density.

The phase continuity equation (1) is modified: it gets a regularizing term and a second order time derivative with small parameters. In [4, 9] some estimations of these parameters are given. Particularly the value $\tau \sim h/c$ is satisfactory (h is the space grid step). The equation type is changed from parabolic to hyperbolic, consequently the three-level explicit scheme with a rather mild stability condition can be used, convective terms are approximated by central differences. In [4, 9] the time step restriction $\Delta t = O(h^{3/2})$ has been justified for this scheme.

As the temperature of all phases and of the rock is identical the system involves (3) as a single equation of the total energy conservation, also modified by analogy with the QGD system [10]. This equation includes the effective coefficient of heat conductivity $\lambda_{\text{eff}} = \varphi \sum_\alpha S_\alpha \lambda_\alpha + (1-\varphi)\,\lambda_r$, and the enthalpy which is calculated via the heat capacity at constant pressure C_{P_α}:

$$H_\alpha = H_{0\alpha} + \int\limits_{T_0}^{T} C_{P_\alpha}(T)\,dT. \tag{6}$$

The connection between the internal energy and the enthalpy is expressed by the next relations:

$$E_\alpha = H_\alpha - \frac{P_\alpha}{\rho_\alpha}, \qquad E_r = H_r. \tag{7}$$

Dependencies of the dynamic viscosity $\mu_\alpha(T)$, the heat capacity $C_{P_\alpha}(T)$ and the coefficient of heat conductivity $\lambda_\alpha(T)$ on temperature are expressed by some empirical relations.

Capillary pressures are described by Parker's functions [11], the relative phase permeability is presented by Stone's Model I [6].

For numerical implementation of the above system an algorithm of the explicit type is proposed. The water pressure P_w, the water saturation S_w, the NAPL saturation S_n and the temperature T are chosen as primary variables in the non-isothermal three-phase case. Main stages of the algorithm are explicit approximations of (1) and (3) to get the auxiliary quantities $(\rho_\alpha S_\alpha)$ and $\big(\varphi \sum_\alpha \rho_\alpha S_\alpha E_\alpha + (1-\varphi)\rho_r E_r\big)$ as well as the solution of a system of nonlinear algebraic equations by Newton's method locally at each computational point to obtain the primary variables [10].

3 Test predictions

For verification of the proposed model and algorithm some problems on three-phase flow in a homogeneous porous medium are considered.

The first one is the problem of phase redistribution under the gravity, this is an infiltration problem. The computational domain is a rectangular reservoir with impermeable boundaries filled by sand. At the initial moment water, oil and gas are distributed over the domain uniformly (see figure 1). Over time, water will go down, gas will rise to the top, and oil will occupy mainly the middle part of the reservoir. Figure 2 demonstrates the comparison of water, oil and gas distributions obtained in isothermal as well as in non-isothermal cases. Infiltration process is accelerated by the temperature gradient, fluid layering occurs more actively. The speed-up and the parallelization efficiency was estimated while computing this test problem on the supercomputer K100 installed at the Keldysh Institute of Applied Mathematics. The computational grid was $200\times200\times100 =$ $= 4$ million points, up to 100 CPU cores were employed. The efficiency achieved was about 90%, the dependence of the speed-up on the number of processors was close to linear.

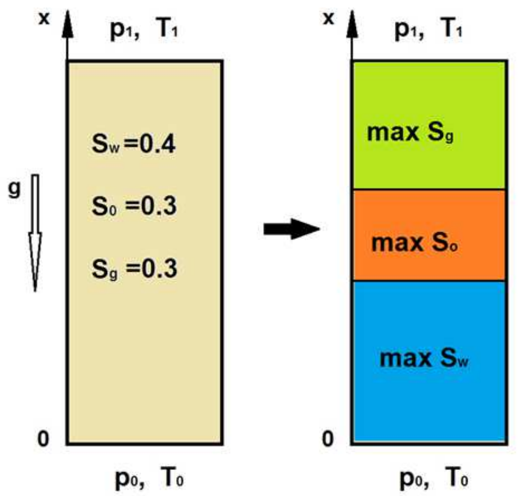
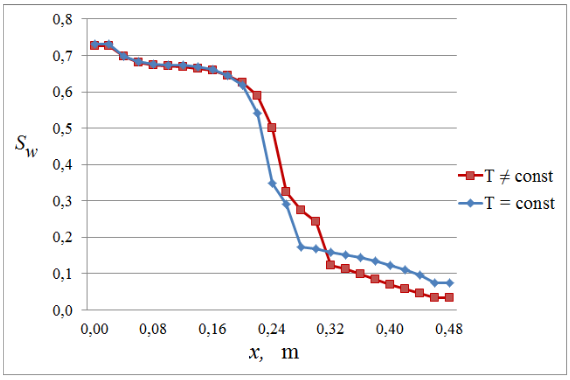
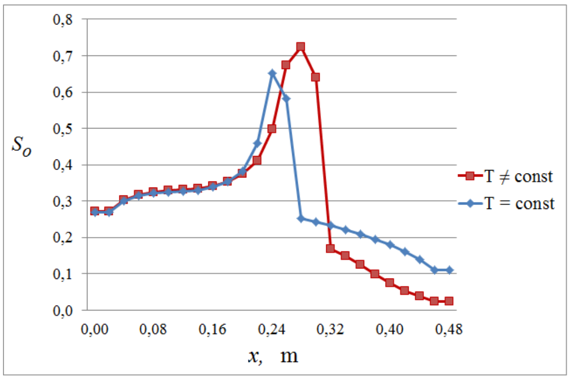


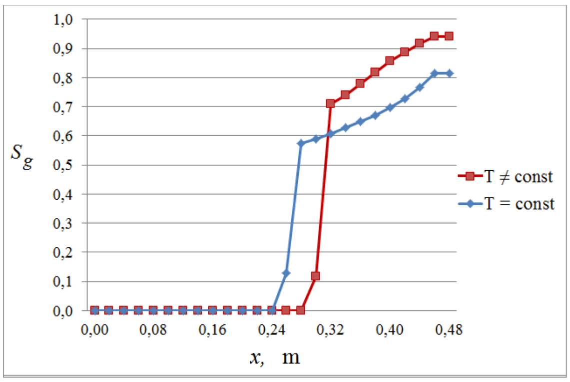
Figure 1 Statement of the problem of phases' redistribution under the gravity



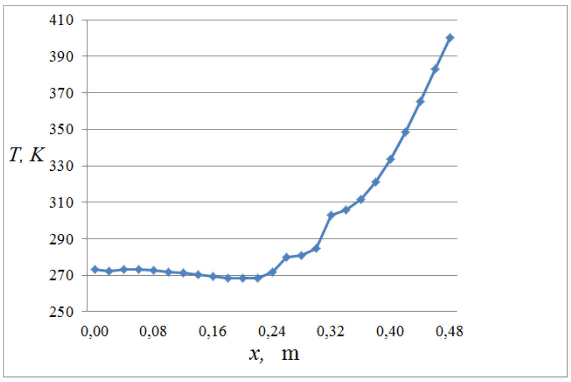
Water saturation



Oil saturation



Gas saturation



Temperature in the non-isothermal case

Figure 2 Results at some time moment for the phases' redistribution problem

The second test is the gas injection problem in a semi-infinite region. The 1D statement is actual: the flow occurs in the horizontal direction from left to right due to the pressure difference at the ends of the considered unit segment, at the left boundary gas is injected under pressure. The initial conditions for the saturation are the uniform distribution as in the previous test, the water pressure equals the atmospheric pressure, the temperature is 285 K. The gas saturation is fixed at the left boundary ($S_g = 0.7$) with the pressure of 1.1 atm.

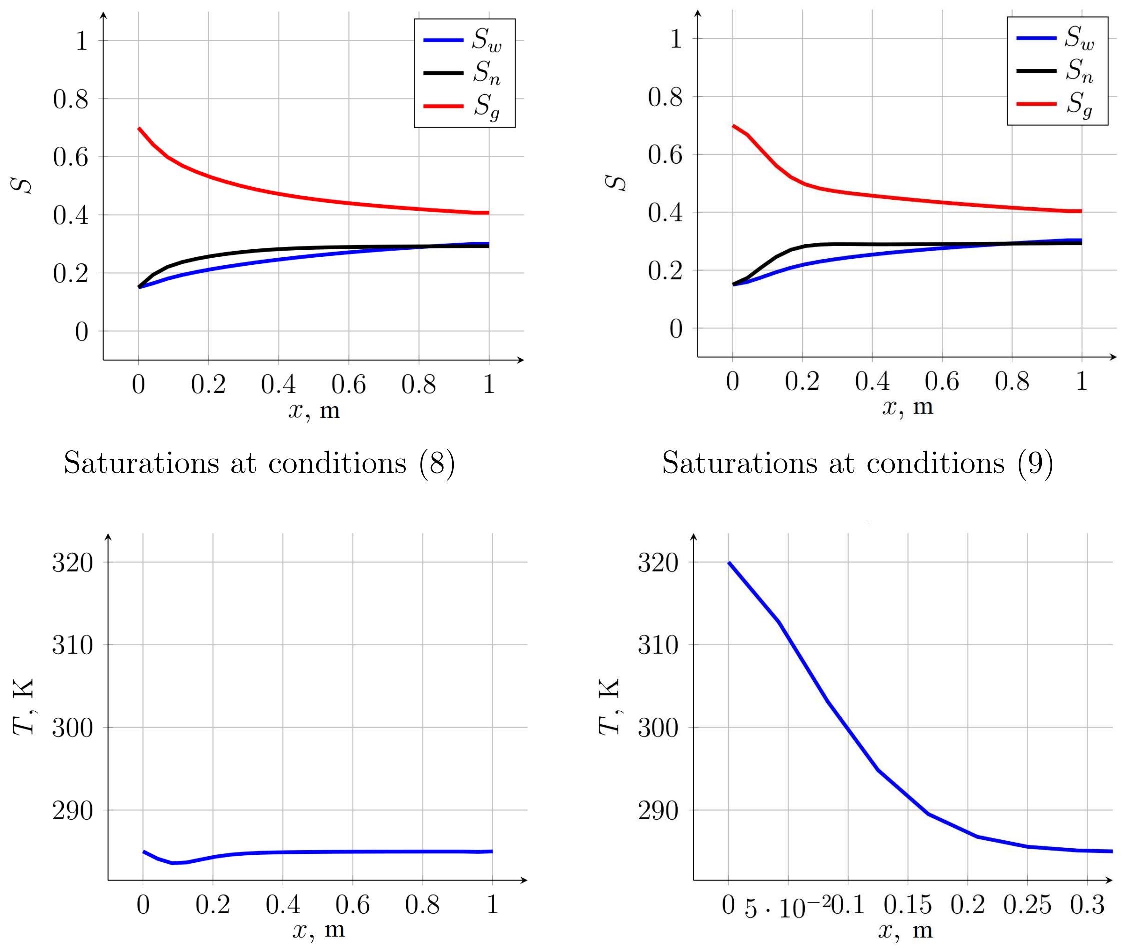
Two variants of the boundary conditions for the temperature are set:

$$T\Big|_{x=0} = T\Big|_{x=1} = 285 \text{ K} \tag{8}$$

or

$$T\Big|_{x=0} = 320 \text{ K}, \quad \frac{\partial T}{\partial x}\Big|_{x=1} = 0. \tag{9}$$

Results of computations at some time moment are depicted in figure 3. The saturation profiles and the temperature distributions under the different conditions for the temperature are compared. Due to the heating the filtration process is faster, the shape of the saturation graphs changes.



Saturations at conditions (8)

Saturations at conditions (9)

Temperature at conditions (8)

Temperature at conditions (9)

Figure 3 Results at some time moment for the gas injection problem

As one more test the 3D problem of infiltration in a cubic domain with a hot source on the top boundary has been solved. Initially in the reservoir the residual water saturation is set, oil and gas are distributed periodically, the water pressure equals the atmospheric pressure, the temperature is 285 K. A source of water with $S_w = 0.6$, $S_n = 0.05$ and the temperature of 320 K at the atmospheric pressure (without pumping) occupies a corner part of the top. The top boundary is open to the atmosphere, the bottom is impermeable, all side faces are permeable. The gravity is taken into account.

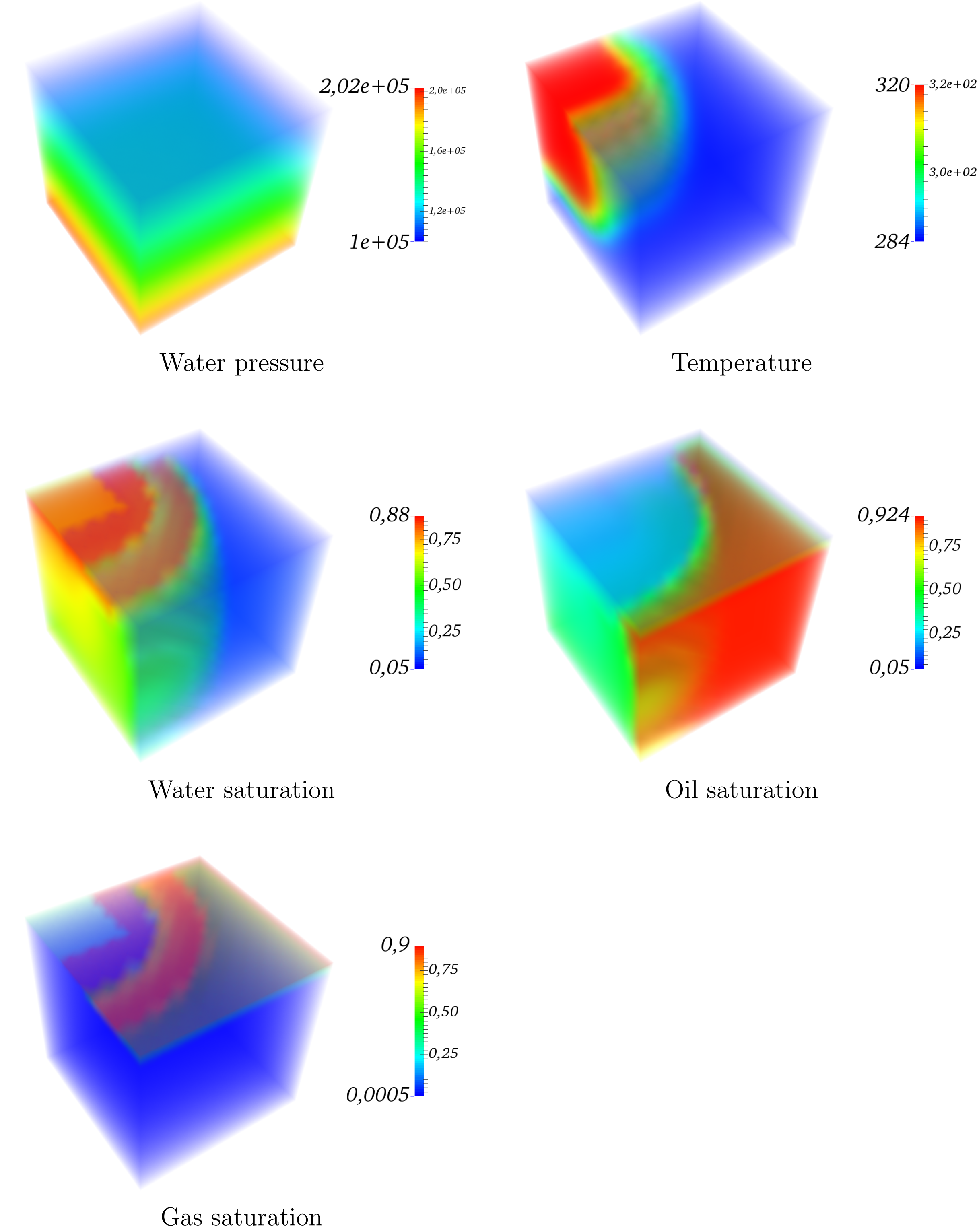


Figure 4 Results for the problem of infiltration in the cubic domain with a hot source on the top boundary

Results at the moment of 1000 seconds are shown in figure 4. One can observe fields of the water pressure, the temperature and the saturation of three phase fluids. Fronts of the temperature and the saturation spread from the source. Water gradually forces out oil and gas and tends to the bottom, the temperature front differs from the water saturation front. Physically correct dynamics of the process is observed.

4 Concluding Remarks

At present the proposed kinetically-based model of multiphase fluid flow in a porous medium is tested on oil recovery problems. The created approach can be used for the simulation of perspective thermal methods of oil recovery (such as heat carrier pumping into the stratum) aimed at increasing the oil production rate of difficult-to-recover hydrocarbon reserves.

References

[1] Bastian, P., J. Kraus, R. Scheichl, and M. Wheel. 2013. *Simulation of flow in porous media: Applications in energy and environment, Radon Series on Computational and Applied Mathematics*. Berlin: De Gruyter. 12. 222 p.

[2] Trapeznikova, M., N. Churbanova, A. Lyupa, and D. Morozov. 2014. Simulation of multiphase flows in the subsurface on GPU-based supercomputers. *Parallel Computing: Accelerating Computational Science and Engineering (CSE), Advances in Parallel Computing*. Amsterdam: IOS Press. 25: 324–333.

[3] Chetverushkin, B. N. 2008. *Kinetic schemes and quasi-gas dynamic system of equations*. Barcelona: CIMNE. 298 p.

[4] Chetverushkin, B. N., D. N. Morozov, M. A. Trapeznikova, N. G. Churbanova, and E. V. Shil'nikov. 2010. An explicit scheme for the solution of the filtration problems. *Mathematical Models and Computer Simulations* 2(6): 669–677.

[5] Chetverushkin, B. N. 2013. Resolution limits of continuous media mode and their mathematical formulations. *Mathematical Models and Computer Simulations* 5(3): 266–279.

[6] Aziz, K., and A. Settari. 1979. *Petroleum reservoir simulation*. London: Applied Science Publ. Lmt. 476 p.

[7] Helmig, R. 1997. *Multiphase flow and transport processes in the subsurface – A contribution to the modeling of hydrosystems*. Berlin: Springer. 367 p.

[8] Pinder, G. F., and W. F. Gray. 2008. *Essentials of multiphase flow and transport in porous media*. John Wiley & Sons. 257 p.

[9] Morozov, D. N., M. A. Trapeznikova, B. N. Chetverushkin, and N. G. Churbanova. 2012. Application of explicit schemes for the simulation of the two phase filtration process. *Mathematical Models and Computer Simulations* 4(1): 62–67.

[10] Lyupa, A. A., D. N. Morozov, M. A. Trapeznikova, B. N. Chetverushkin, and N. G. Churbanova. 2014. Three-Phase Filtration Modeling by Explicit Methods on Hybrid Computer Systems. *Mathematical Models and Computer Simulations* 6(6): 551–559.

[11] Parker, J. C., R. J. Lenhard, and T. Kuppusami. 1987. A parametric model for constitutive properties governing multiphase flow in porous media. *Water Resources Research* 23(4): 618–624.

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МОДЕЛИРОВАНИЕ НЕИЗОТЕРМИЧЕСКИХ МНОГОФАЗНЫХ ТЕЧЕНИЙ В ПОРИСТЫХ СРЕДАХ С ИСПОЛЬЗОВАНИЕМ ЯВНЫХ РАЗНОСТНЫХ СХЕМ*

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Статья посвящена разработке алгоритмов и программ для моделирования сложных течений жидкости в подземном пространстве. Эти средства могут быть использованы в важных практических приложениях, таких как проблемы добычи нефти и газа, экологические проблемы, связанные с загрязнением почвы и грунтовых вод, и многих других. Основной целью исследования является реализация математической модели фильтрации явными численными методами с достаточно мягкими условиями устойчивости, чтобы обеспечить экономичность расчетов на современных суперкомпьютерах при высокой эффективности распараллеливания.

Для достижения этой цели классическая модель многофазного течения слабосжимаемой жидкости в пористой среде была модифицирована по аналогии с квазигазодинамической системой уравнений, также была проведена гиперболизация полученной системы. В результате уравнение неразрывности фазы приобрело дополнительные члены с малыми параметрами — регуляризатор и вторую производную по времени. Соответствующая трехслойная явная разностная схема имеет второй порядок аппроксимации по времени и по пространству. Модель учитывает возможные источники тепла, гравитационные и капиллярные силы.

Предложенный подход верифицирован с помощью ряда тестовых расчетов, численно получены физически корректные результаты. Достигнуто высокое ускорение вычислений на гибридных кластерах, включающих многоядерные центральные процессоры (CPU) и графические ускорители (GPU).

Ключевые слова: пористая среда, многофазное течение жидкости, квазигазодинамическая система, явные разностные схемы, параллельные вычисления

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Литература

[1] *Bastian P., Kraus J., Scheichl R., Wheel M.* Simulation of flow in porous media: Applications in energy and environment // Radon Series on Computational and Applied Mathematics. – Berlin: De Gruyter, 2013. Vol. 12. 222 p.

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[2] *Trapeznikova M., Churbanova N., Lyupa A., Morozov D.* Simulation of multiphase flows in the subsurface on GPU-based supercomputers // Parallel Computing: Accelerating Computational Science and Engineering (CSE), Advances in Parallel Computing. – Amsterdam: IOS Press, 2014. Vol. 25. P. 324–333.

[3] *Chetverushkin B. N.* Kinetic schemes and quasi-gas dynamic system of equations. Barcelona: CIMNE, 2008. 298 p.

[4] *Chetverushkin B. N., Morozov D. N., Trapeznikova M. A., Churbanova N. G., Shil'nikov E. V.* An explicit scheme for the solution of the filtration problems // Mathematical Models and Computer Simulations, 2010. Vol. 2(6). P. 669–677.

[5] *Chetverushkin B. N.* Resolution limits of continuous media mode and their mathematical formulations // Mathematical Models and Computer Simulations, 2013. Vol. 5(3). P. 266–279.

[6] *Aziz K., Settari A.* Petroleum reservoir simulation. London: Applied Science Publ. Lmt., 1979. 476 p.

[7] *Helmig R.* Multiphase flow and transport processes in the subsurface – A contribution to the modeling of hydrosystems. Berlin: Springer, 1997. 367 p.

[8] *Pinder G. F., Gray W. F.* Essentials of multiphase flow and transport in porous media. John Wiley & Sons, 2008. 257 p.

[9] *Morozov D. N., Trapeznikova M. A., Chetverushkin B. N., Churbanova N. G.* Application of explicit schemes for the simulation of the two phase filtration process // Mathematical Models and Computer Simulations, 2012. Vol. 4(1). P. 62–67.

[10] *Lyupa A. A., Morozov D. N., Trapeznikova M. A., Chetverushkin B. N., Churbanova N. G.* Three-phase filtration modeling by explicit methods on hybrid computer systems // Mathematical Models and Computer Simulations, 2014. Vol. 6(6). P. 551–559.

[11] *Parker J. C., Lenhard R. J., Kuppusami T.* A parametric model for constitutive properties governing multiphase flow in porous media // Water Resources Research, 1987. Vol. 23(4). P. 618–624.

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